20th WCNDT 20th World Conference on Non-Destructive Testing
Songdo Convensia, Incheon, Korea, 27~31 May 2024

Potentials of Quantum Computing and First Applications in Computed Tomography

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Abstract: Quantum Computing (QC) technology has obtained a lot of attention recently. First QC-devices
are available as test beds for real-world applications. In this field, the Fraunhofer Gesellschaft has started a close collaboration with IBM in 2020. Since then, Fraunhofer Development Center
X-ray Technology (EZRT) has access to a real-world NISQ-device (Noisy Intermediate Scale Quantum), although the number of available qubits is still low (typically less than 100).
Basically, there are two types of QC-devices, the above mentioned NISQ-systems and the class
of Quantum Annealers which are restricted mostly to optimization problems, but today already
provide several thousands of qubits with each device. On the other hand, the NISQ-systems provide a certain set of "commands", called quantum gates, which can be combined in any arbitrary way to form an elaborated quantum circuit. A quantum circuit may be seen as an analog to a program, although there are decisive differences. The Fraunhofer EZRT is involved in three projects concerning quantum computing, dealing with
general research questions in this field, but also specifically with industrial research activities. In them, we research and apply existing quantum computing solutions to numerical problems as they often occur in the field of X-ray Computed Tomography (CT), as well as experimentally
determine practical limits to the existing theoretical works.
In our talk we review the current state of QC-technology and give a short overview of the theoretical background. The concept of qubits, superposition, entanglement and measurement of quantum states will be explained. Quantum gates and circuits built out of these gates will be
introduced. In the second part, we will report on the current state of our research. The three running
projects include an implementation of cross-sectional image reconstruction for CT imaging based on a QUBO-method (Quadratic Unconstrained Binary Optimization), used to find a mathematical minimum of a metric.
Another topic, we are working on, is optimization of X-ray source trajectory in CT with respect to image quality achievable by a restricted number of angular positions. In addition, we evaluate common problems of image processing for instance noise reduction and search for ways to implement respective methods on the QC. Acknowledgement
The mentioned quantum projects are funded by the Bavarian Ministry of Economic Affairs, Regional Development and Energy.

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Overview

Computed Tomography and Quantum Computing

Introduction (1)

- **Principles of Quantum Computing (QC)**
- **Programming a QC-device**

Introduction (2)

- Principles of Computed Tomography (CT)

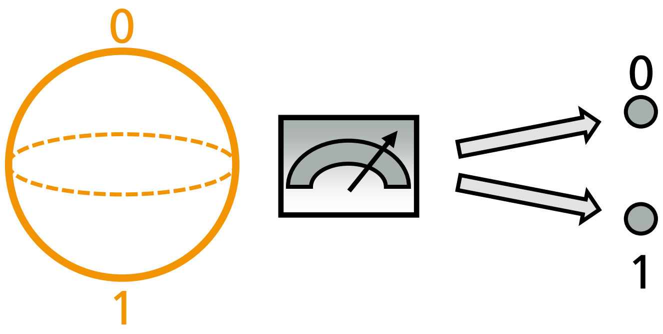
Putting things together

- QC-based optimization of trajectories
- QC-based Image reconstruction

Conclusions & Outlook

QC theory

What is superposition and what does a measurement?



Measurement in quantum physics:

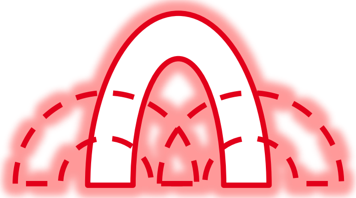
- When a qubit is read out (measured), the result is 0 or 1
- Coefficients c_0 und c_1 in $\binom{c_0}{c_1}$ represent the probability:
  - Probability for measuring 0: $|c_0|^2$
  - Probability for measuring 1: $|c_1|^2$
- Normalization: $|c_0|^2 + |c_1|^2 = 1$
- After measurement the qubit is in the measured state (called collapse of the wave function)

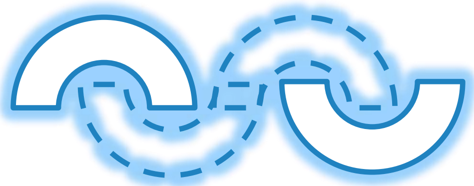
QC theory

Similarity of classical and quantum computers

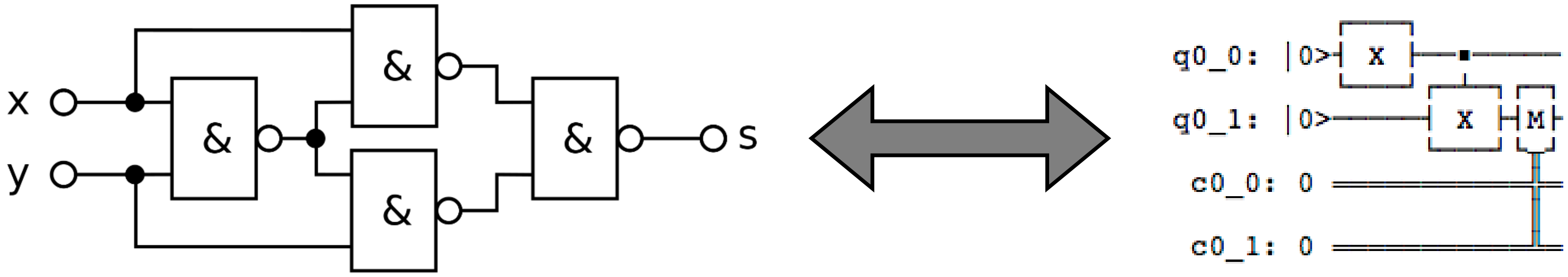
- Specific input state
  QC: with superposition

- Concatenation of gates
  QC: Qubits get manipulated and connected to each other
  → interference and entanglement





- Read (qu)bit by measurement
  QC: several measurements are necessary because result depends on probability



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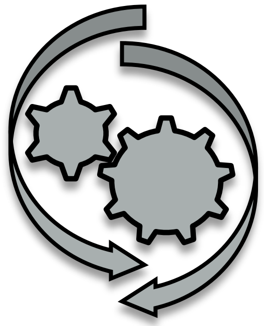
QC theory

Differences between classical and quantum computers

QC gates must be invertible

As many outputs as inputs

Mathematical: matrix which describes gate must be unitary



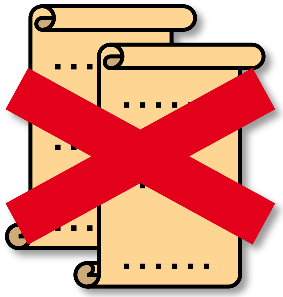
Qubit states cannot be copied (no-cloning-theorem)

„result state“ at the end of a QC circuit cannot be copied

Problem: state is „destroyed“ by measurement (wave collapse)

Several executions are needed to get the distribution of the result

→ 100 up to 1000 “shots”, histogram of probability for all possible results



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Tensor product

A **quantum register** is a combination of several qubits

→ mathematically denoted as tensor product: ⊗

$$\begin{pmatrix}\beta_0\\ \beta_1\end{pmatrix} \otimes \begin{pmatrix}\gamma_0\\ \gamma_1\end{pmatrix} = \begin{pmatrix}\beta_0\gamma_0\\ \beta_0\gamma_1\\ \beta_1\gamma_0\\ \beta_1\gamma_1\end{pmatrix}$$

Analogous **quantum gates**, example Hadamard:

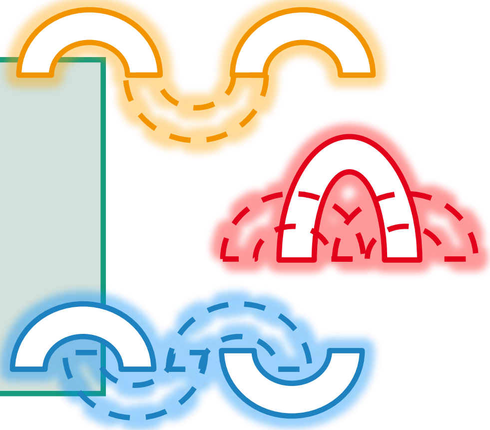
$$H \otimes H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \otimes \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1\\ 1 & 1 & -1 & -1\\ 1 & -1 & -1 & 1\end{bmatrix}$$

QC – basic facts

What does make quantum computers so powerful?

Why is QC (sometimes) more powerful than classical computing?

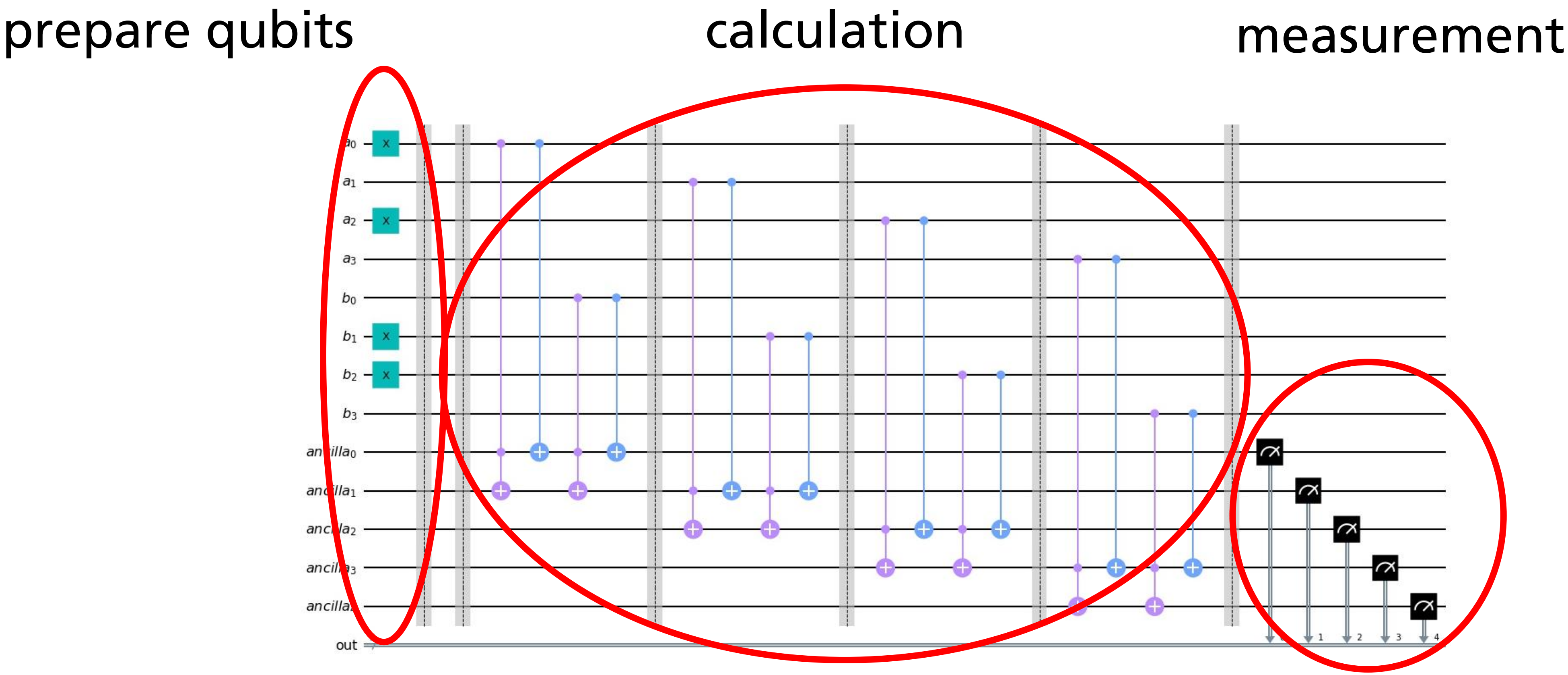
- Use of quantum mechanical properties:
  superposition, **interference** and **entanglement**
- QC can be faster than classical computers, but not in general
- Special algorithms are needed, which are difficult to develop



Example: Memory usage for the simulation of a quantum computer

- Representation of one qubit as two complex numbers represented as four real 8 byte numbers
- Memory usage for one qubit: $4 \cdot 8\ \text{byte} = 2^5\text{byte}$
- Each additional qubit doubles the memory usage!
- Simulation of a QC with 45 qubits:
  Memory usage: $2^{44} \cdot 2^5\ \text{byte} = 5.62949953421 \cdot 10^{14}\ \text{byte} =\ \mathbf{0.5\ petabyte}$

How to write a „program“ on a QC-device?



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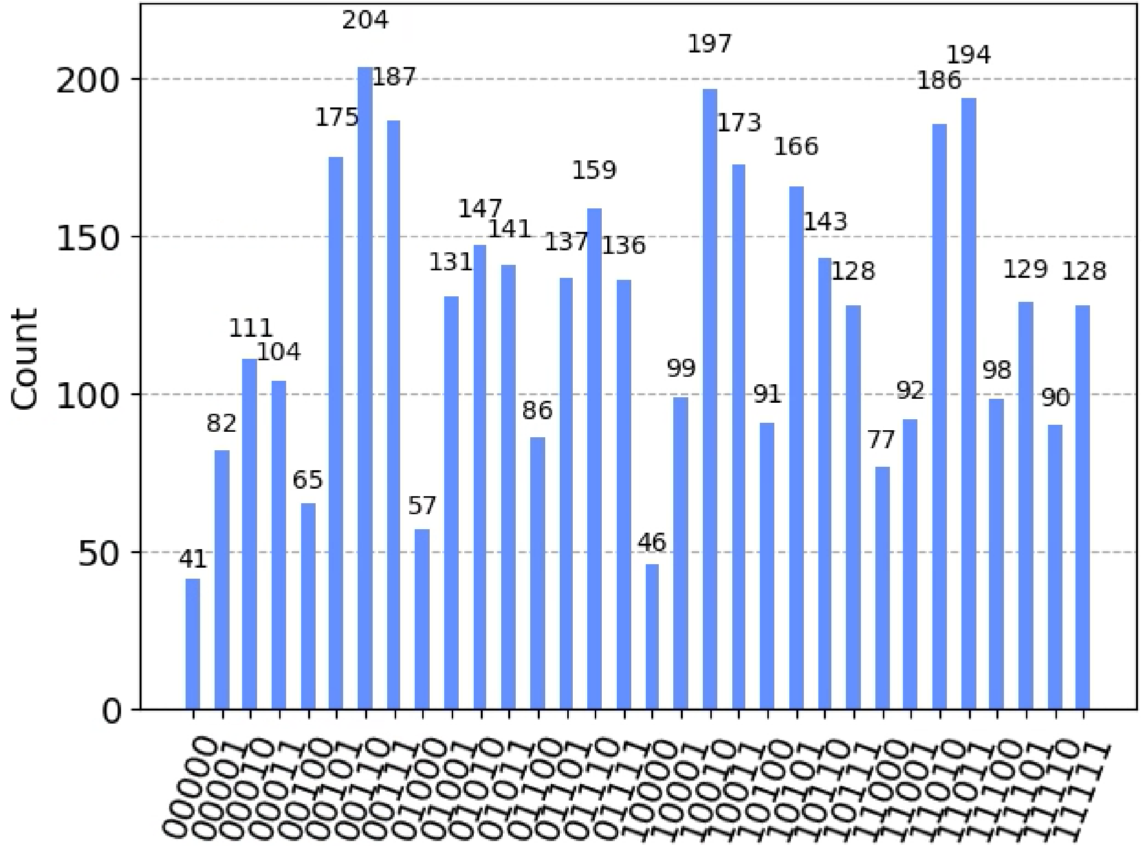
How to retrieve results from a QC-device?

→ INTERPRETING HISTOGRAMS

Repeat execution of quantum circuit many times

Typical 100 … 1000 “shots”

Resulting in a Histogram showing the frequency of occurrence of all possible states



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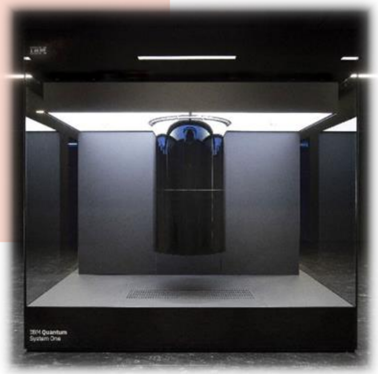
Quantum Computing – some basic facts

Types of QC hardware

Universal Quantum Computer	Quantum Annealer

• arbitrary quantum circuits by stringing together gates that manipulate qubits (quantum bits)
• various realizations:
  • ion traps: Manipulation of single ions = qubit (encoding via energy states)
  • optical systems: photon = qubit (coding e.g. via polarization)
  • superconductors with a Josephson contact
  • and other technologies…
	• lattice-like networking of qubits
	• qubits cannot be manipulated individually
	• can only solve special problems → not a universal computer
	• solution of minimization problems → problem must be formulated in this way
	• best known quantum annealer: D-Wave (qubits based on superconductivity)



> 127 qubits

> 5000 qubits

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Overview

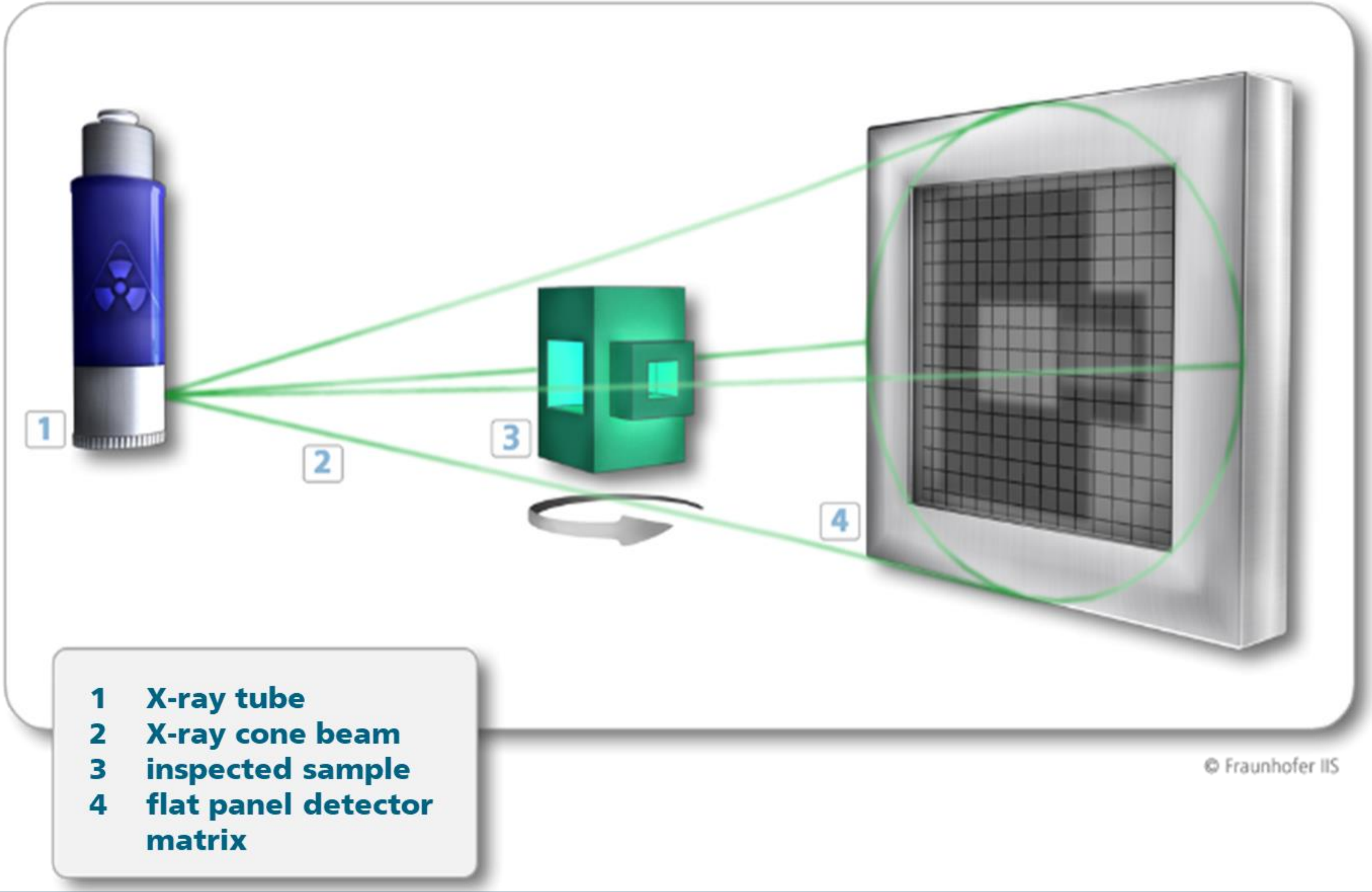
Computed Tomography and Quantum Computing

Optimization of CT Data Acquisition by means of Quantum Computing

Principles of Computed Tomography Imaging

Conventional data acquisition process

- move X-ray source on a circular path around object
- equivalent: rotate object between X-ray source and flat panel sensor
- complete 360° data set
- simple parametrization
- drawback: time consuming, redundant information



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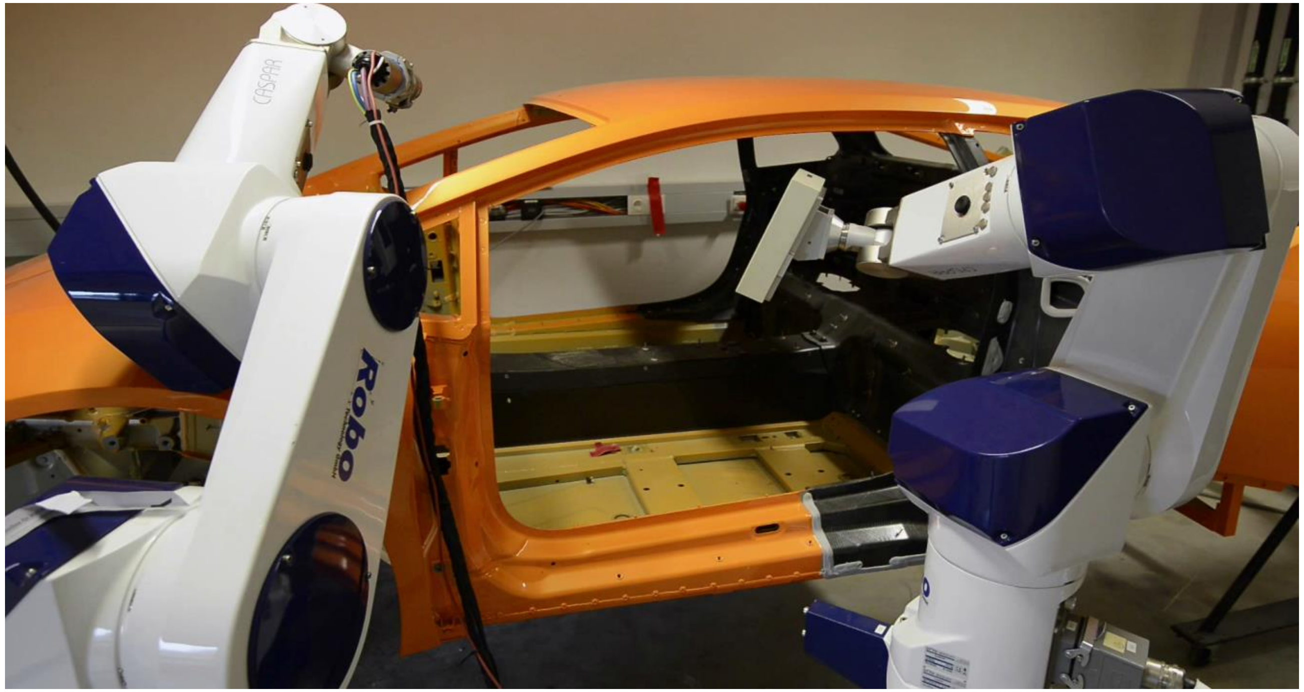
Optimization of CT Data Acquisition by means of Quantum Computing

Optimizing Trajectories

Why using non-circular trajectories?

- move x-ray source and detector array independently by robots
- improve access to complex and / or very large parts
- save time & money by accessing only the "most important" projections, e.g. reducing from 1000 to 5000 views to 100 to 200
- avoid artifacts by eliminating problematic directions



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- **QC-based Image reconstruction**

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Optimization of CT Data Acquisition by means of Quantum Computing

Optimizing Trajectories for Computed Tomography

Formulation of the optimization problem

- **vector $\vec{p}$ is defined for each projection:**
  - direction encodes angular position of the X-ray source (from where the projection is created)
  - length depicts the edge quality of the object from that direction
- **the basic idea is, do NOT search for the *n* "best" out of *N* projections, but for the "best" set of *n* projections**
- **just looking at single projection by measuring their information content, will lead to very similar data from close-by positions**
- **several metrics were developed and tested**

$$\vec{p}_i = \sum_{j=1}^{J} c_{j,i}^{max} \cdot \frac{(\vec{d}_{j,i} - \vec{s}_i)}{\left|\vec{d}_{j,i} - \vec{s}_i\right|}$$

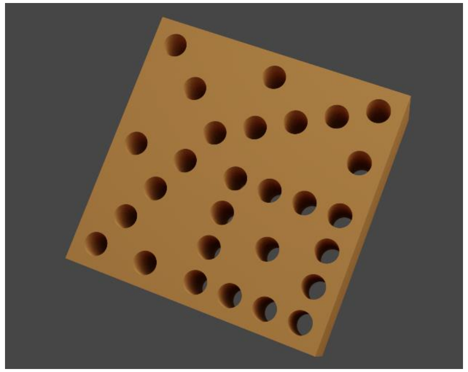
$c_{j,i}^{max}$: **max. wavelet coefficient**

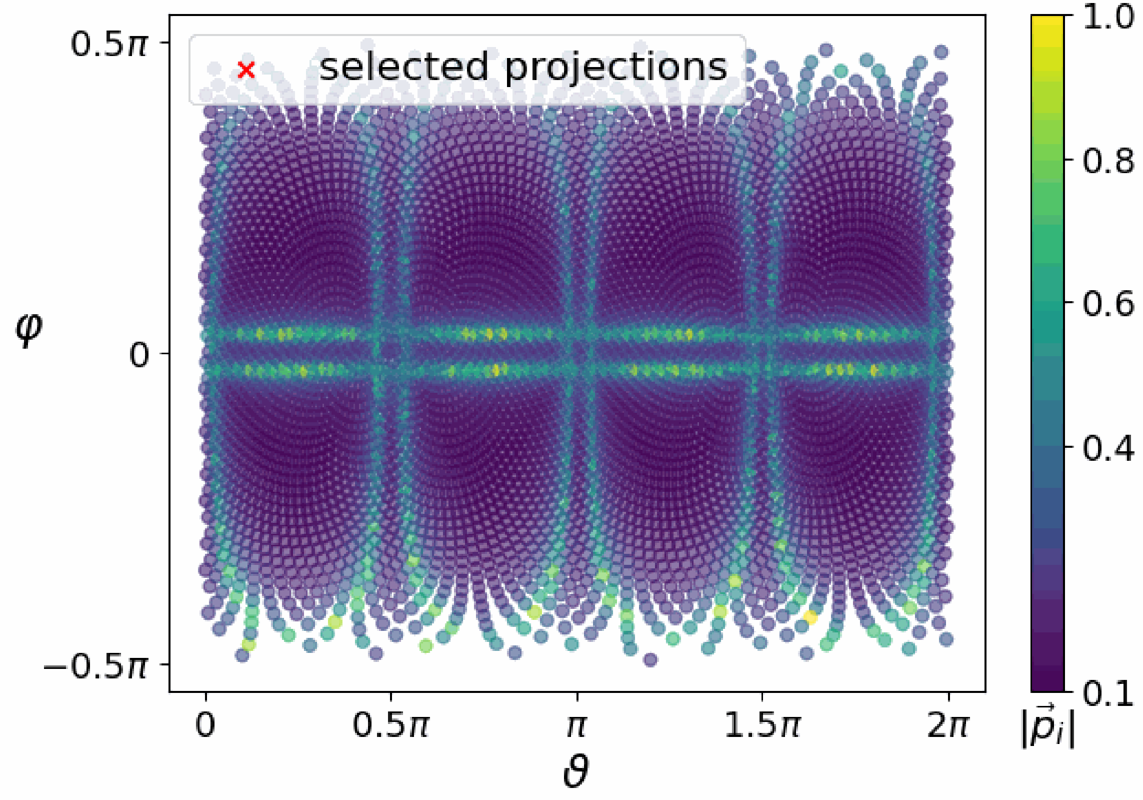
$\vec{d}_{j,i}$: **corresponding pixel position**

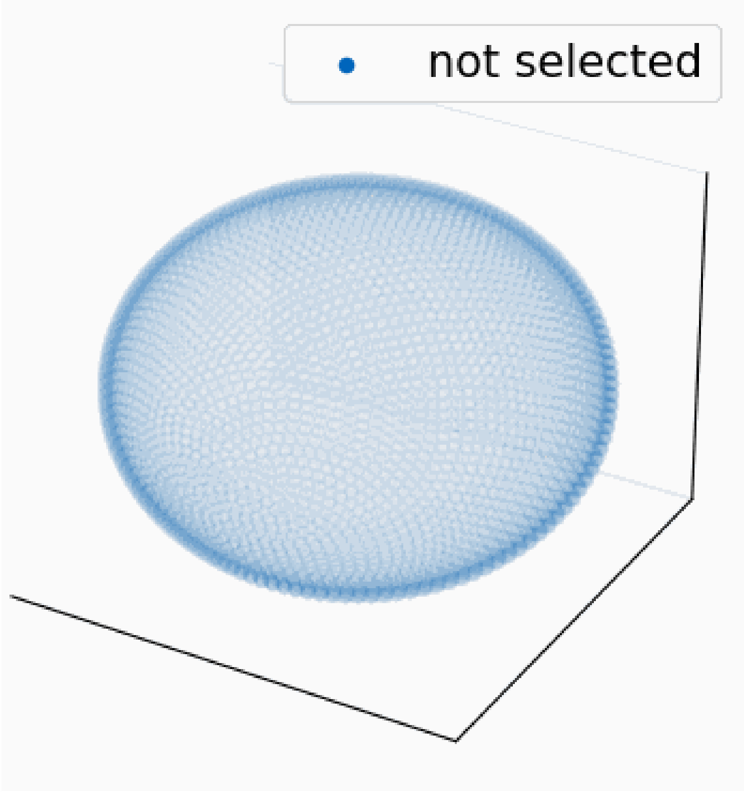
$\vec{s}_i$: **source position**

Partial metric 1: Reducing redundancy due to neighboured projections

Optimizing Trajectories for Computed Tomography







→ **maximization of edge information from many directions**

Quantum Computing – well known applications

Optimization by QC

Use Quantum Annealer to solve combinatory binary optimization problems

- these mathematical problems include:
  - **Qu**adratic **B**inary **O**ptimization
  - Ising-models
  - graph theory

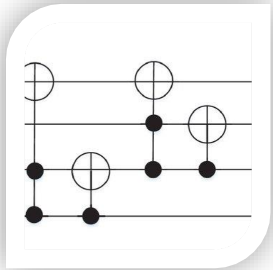
- the approaches can be transformed into each other:
  - graph problems can be reformulated as QUBOs
  - with $s = 2x - 1$ the QUBO becomes an Ising-model

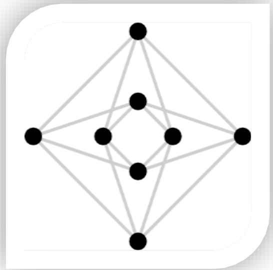
QUBO $$f_Q(x) = \sum_{i=1}^{n} \sum_{j=1}^{i} q_{ij} x_i x_j$$

$$\boldsymbol{x_i} = \{\mathbf{0}, \mathbf{1}\}$$

Ising-model $$\hat{\mathcal{H}} = -\frac{1}{2} \sum_{i,j} J_{ij} s_i^z s_j^z - H_z \sum_{i=1}^{N} s_i^z$$

$$s_i^z = \pm 1;$$





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Optimization of CT Data Acquisition by means of Quantum Computing

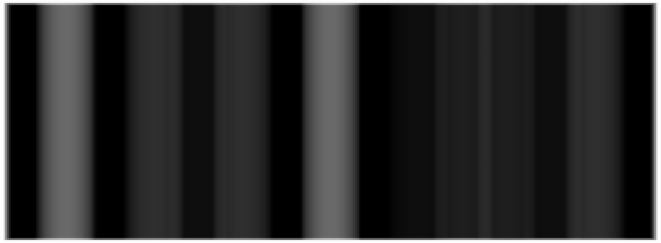
Putting everything together

- **set-up model for X-ray measurement**
- **test object: perforated plate**
- **calculate virtual projections**
- **define measure to quantify „amount of information" in each projection**
- **combine 2 metrics**

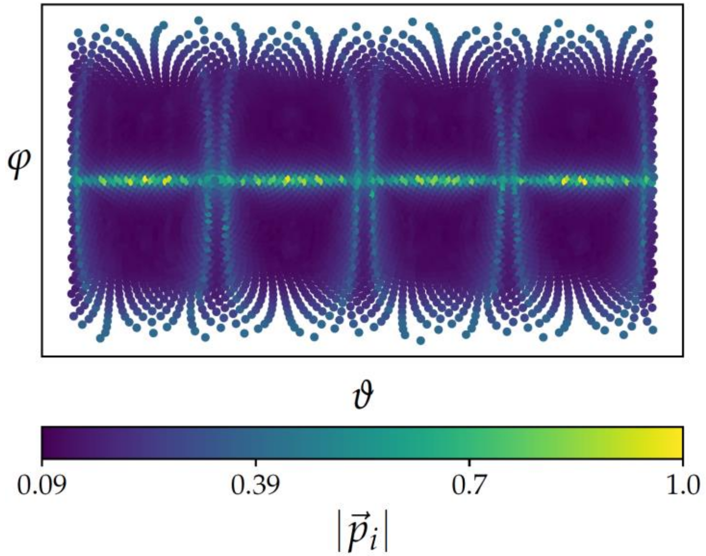
$$f_{M1}(x) = \sum_{k}^{3} \left\| \sum^{N} p_{k,i}\, x_i \right\|^2$$

$$f_{M2}(x) = \sum_{i,j}^{N} V_{i,j} x_i x_j$$

$$f_Q(x) = f_{M1}(x) + f_{M2}(x),$$



simulated projection of the plate at ϕ = 0° and ϑ = 0° (w/o photon noise)



map of quality measure ***p*** for 5,000 projection equally distributed on a 4π solid angle

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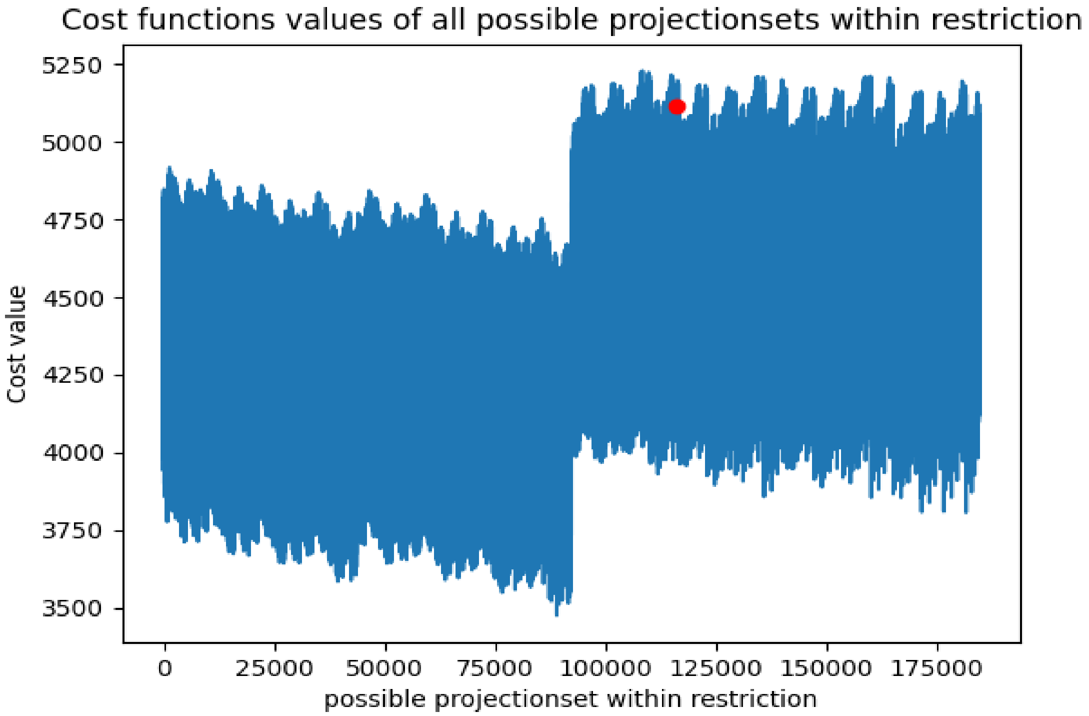
Optimization of CT Data Acquisition by means of Quantum Computing

Results

Cost function values of all possible projection sets

- $x_i = 1$ means projection p_i is taken into account for reconstruction, else not ($x_i = 0$)
- Chose 10 out of 20 projection sets
- there are $\binom{20}{10} = 184756$ possible combinations
- the ordinate shows a label given to all sets of projections taken into account → variable x
- the red dot marks the result from the QC



$\{x_1, \dots, x_{20}\}$

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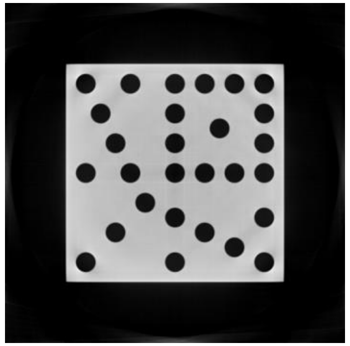
Optimization of CT Data Acquisition by means of Quantum Computing

Results

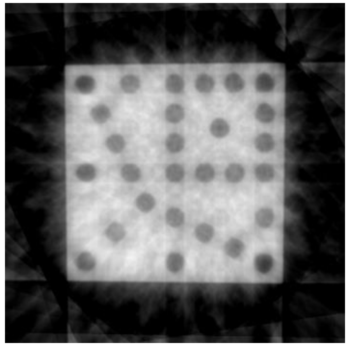
Cross-sectional images of test body

- ground truth vs. considerably reduced number of projections
- reconstruction from 10 selected projections on a semi-circular trajectory

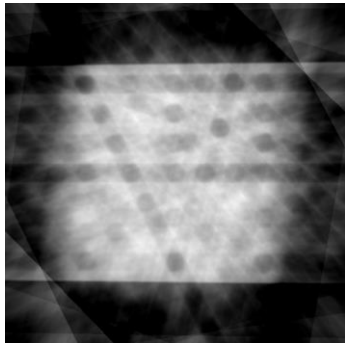
1,000 equidistant projections



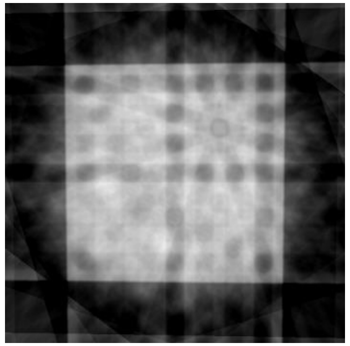
20 equidistant projections along a semi-circular trajectory



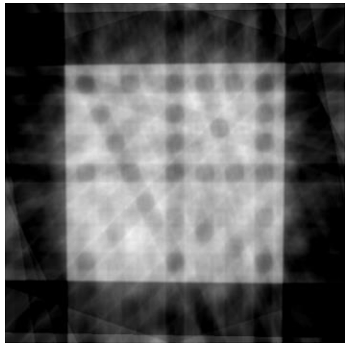
10 randomly sampled projections



10 equidistant projections



projection set from QC



QUBO based Ansatz for CT image reconstruction

Working on a discrete representation of the data

Formulation of the task

- Reconstruction problem = inverse problem $\mathrm{y} = \mathrm{Mx}$

y ➔ measured data (projections)

M ➔ matrix describing the CT system

x ➔ reconstructed volume with L pixels

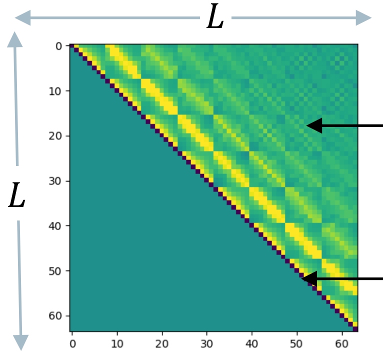
- Least squares formulation:

$$H(\mathrm{x}) = \|\mathrm{Mx} - \mathrm{y}\|_2^2 = \underbrace{\mathrm{x}^T\mathrm{M}^T\mathrm{Mx}}_{\text{quadratic in } \mathbf{x}} - \underbrace{\mathrm{x}^T\mathrm{M}^T\mathrm{y} - \mathrm{y}^T\mathrm{Mx}}_{\text{linear in } \mathbf{x}} + \underbrace{\mathrm{y}^T\mathrm{y}}_{\text{constant}}$$

$$\mathrm{Q}_{i,j} = \sum_k \mathrm{M}_{ki}\mathrm{M}_{kj} \qquad \mathrm{Q}_{i,i} = -2\sum_j \mathrm{y}_j\mathrm{M}_{ji}$$

negligible

- QUBO size: L^2



Nau MA et al, Exploring the Limitations of Hybrid Adiabatic Quantum Computing for Emission Tomography Reconstruction. *Journal of Imaging*. 2023

QUBO based Ansatz for CT image reconstruction

Multi bit representation

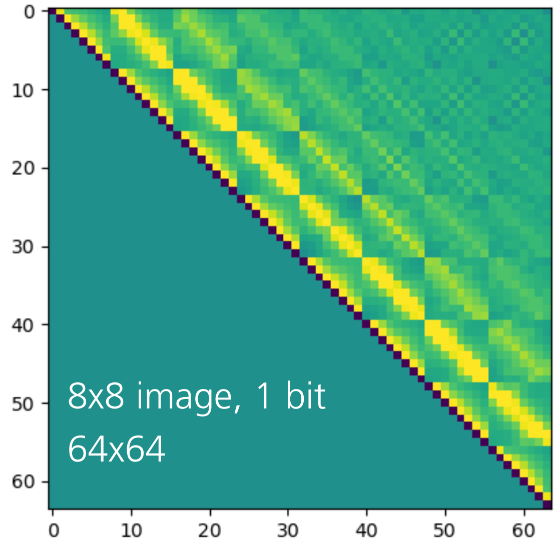
- Replace variable x_i for n-bit-representation by n binary variables

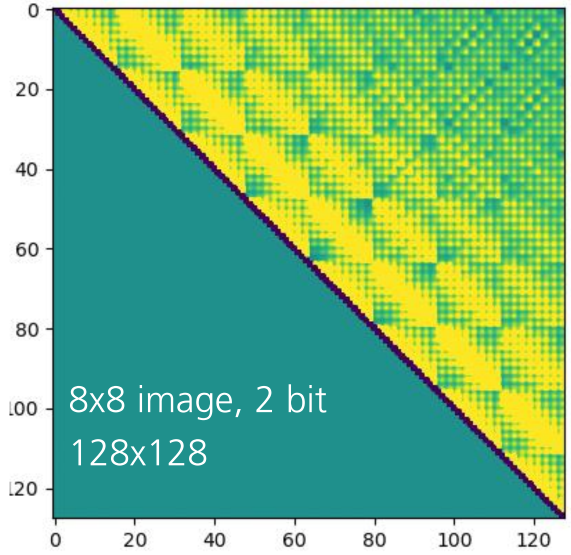
  $x_{i,0}, x_{i,1}, \dots x_{i,n-1}$ in binary representation
- Size of the QUBO matrix $(L * n)^2$
- Examples for different dimensions:
  - Reconstruction 64 by 64 pixels @ 1 bit

    → 16,7 Mio. combinations
  - Reconstruction 64 by 64 pixels @ 4 bit

    → 267 Mio. combinations
  - Reconstruction 1024 by 1024 pixels @ 4 bit

    → $17{,}6 * 10^{12}$ combinations





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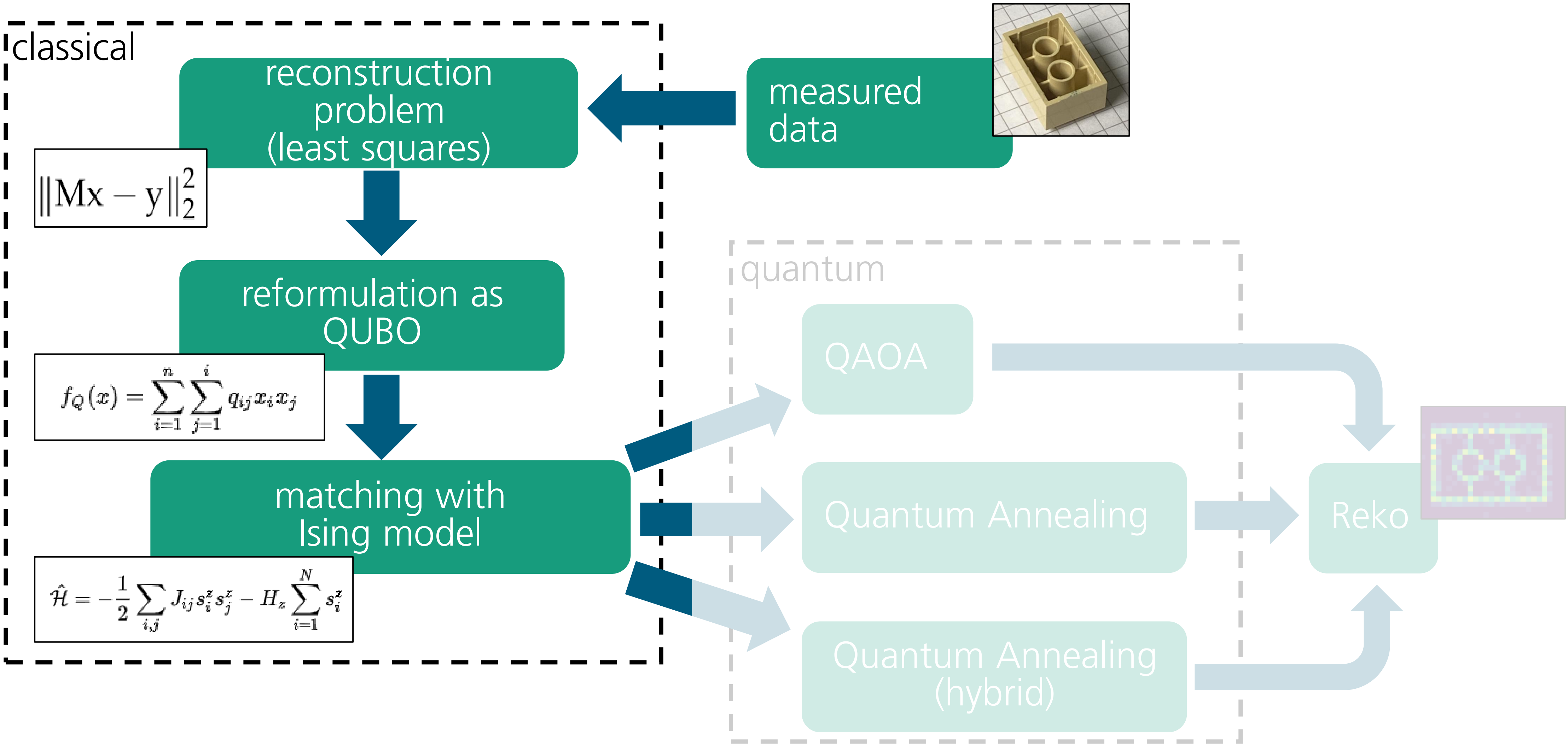
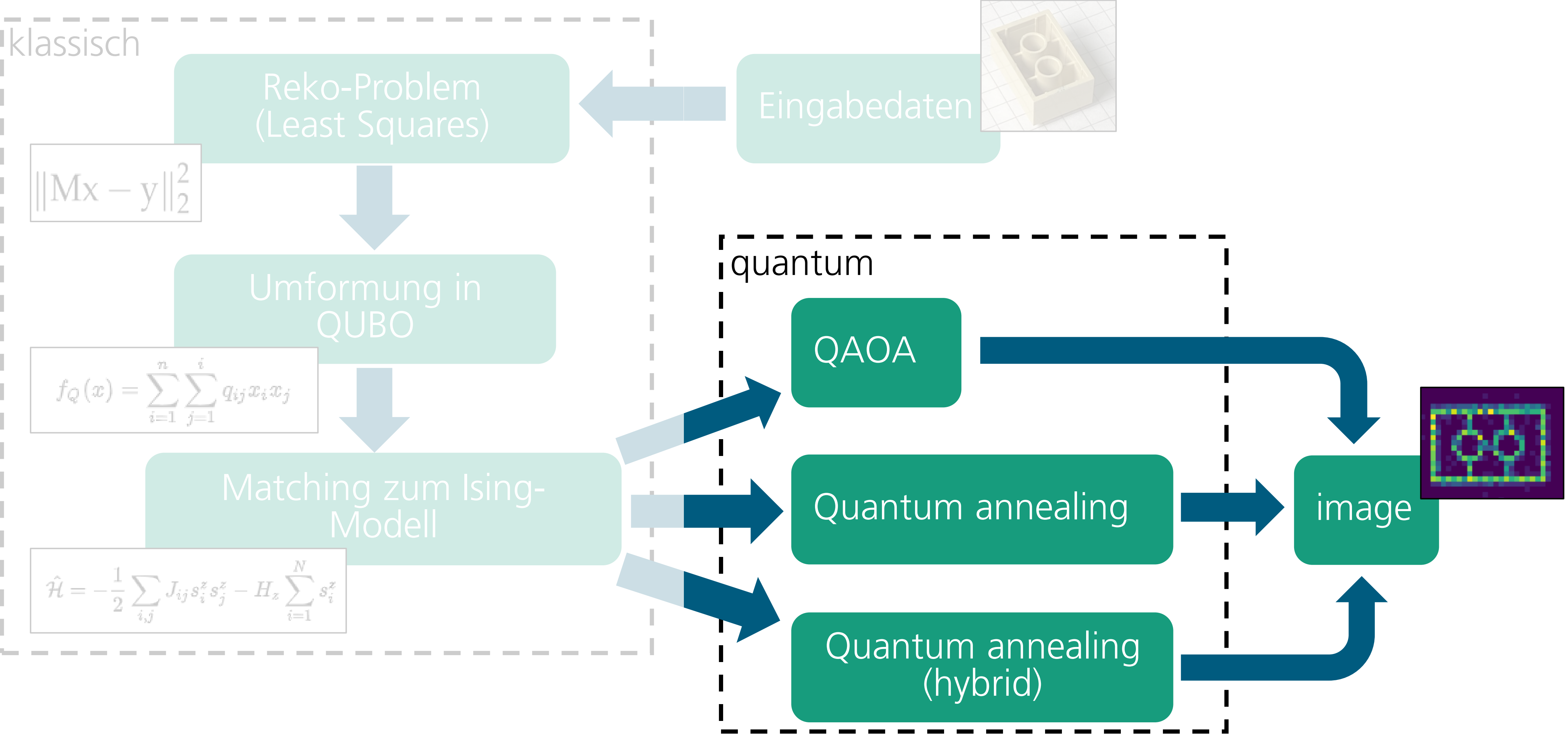
QUBO based Ansatz of CT image reconstruction



QUBO based Ansatz of CT image reconstruction



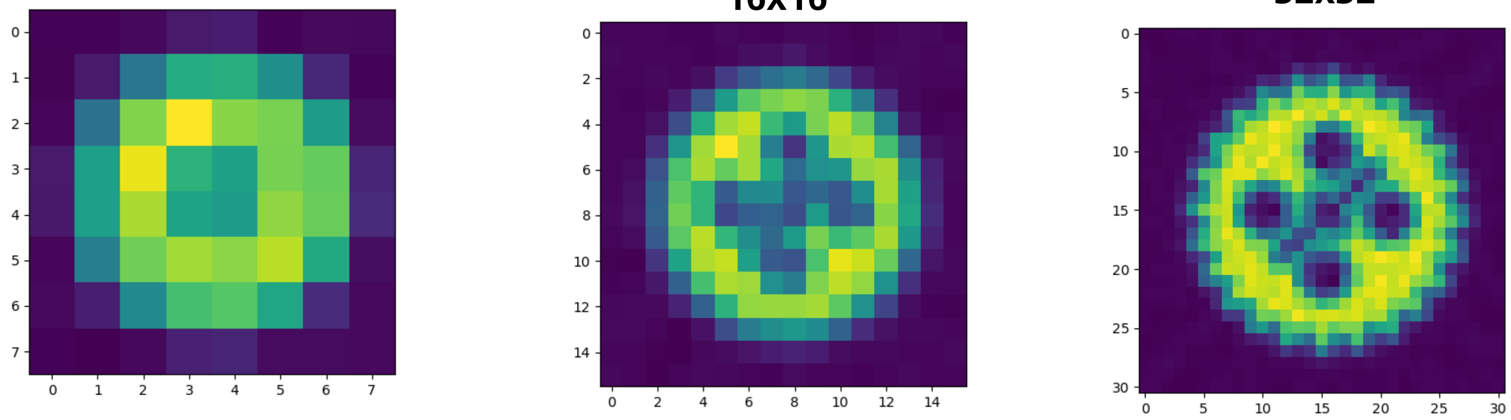
Working on real (measured) data

real data – conventional filtered back projection FBP

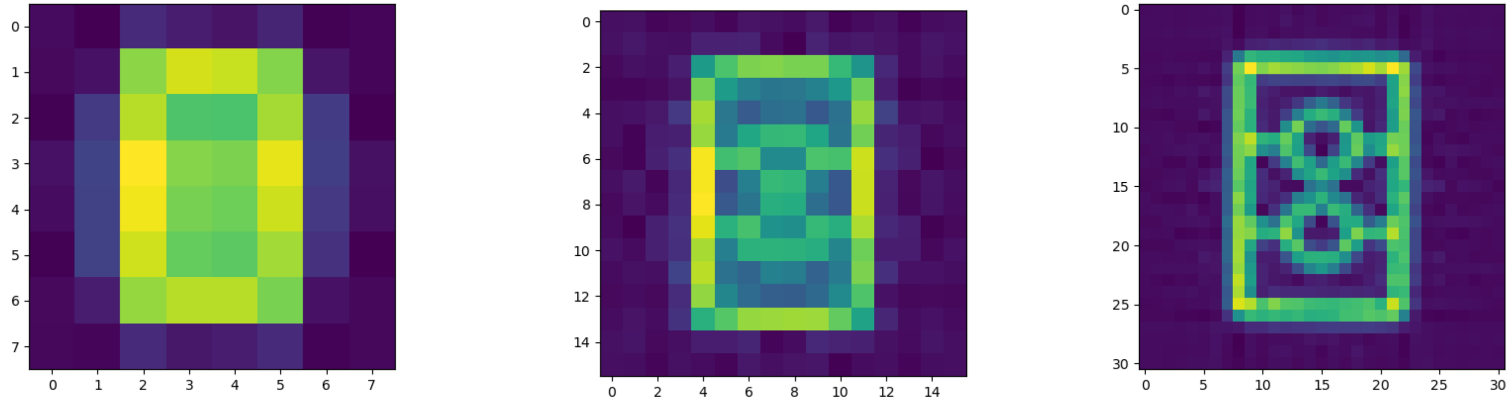
- µ-CT, 60 kV
- #projections 1200
- angular range: 360°

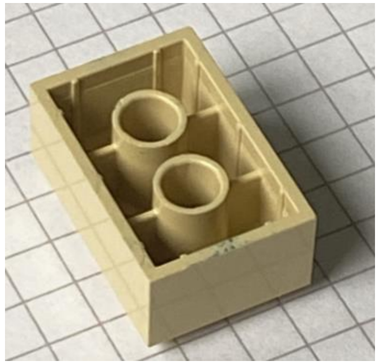
8x8 **16x16** **32x32**

Lego toothed wheel



Lego brick





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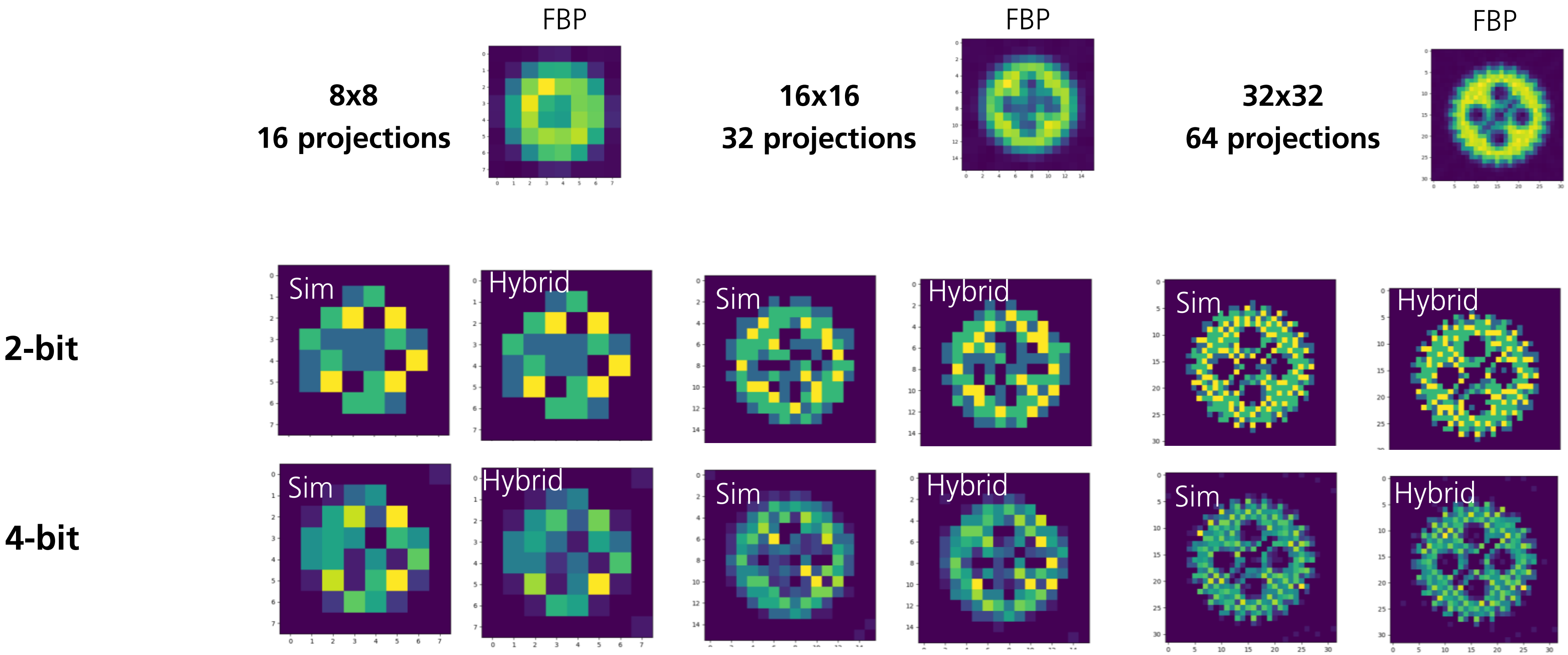
Working on real (measured) data

Simulated annealing and hybrid quantum annealing

Lego toothed wheel



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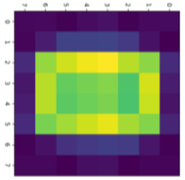
Working on real (measured) data

Simulated annealing and hybrid quantum annealing

Lego brick

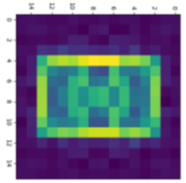
8x8
16 projections

FBP



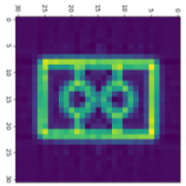
16x16
32 projections

FBP



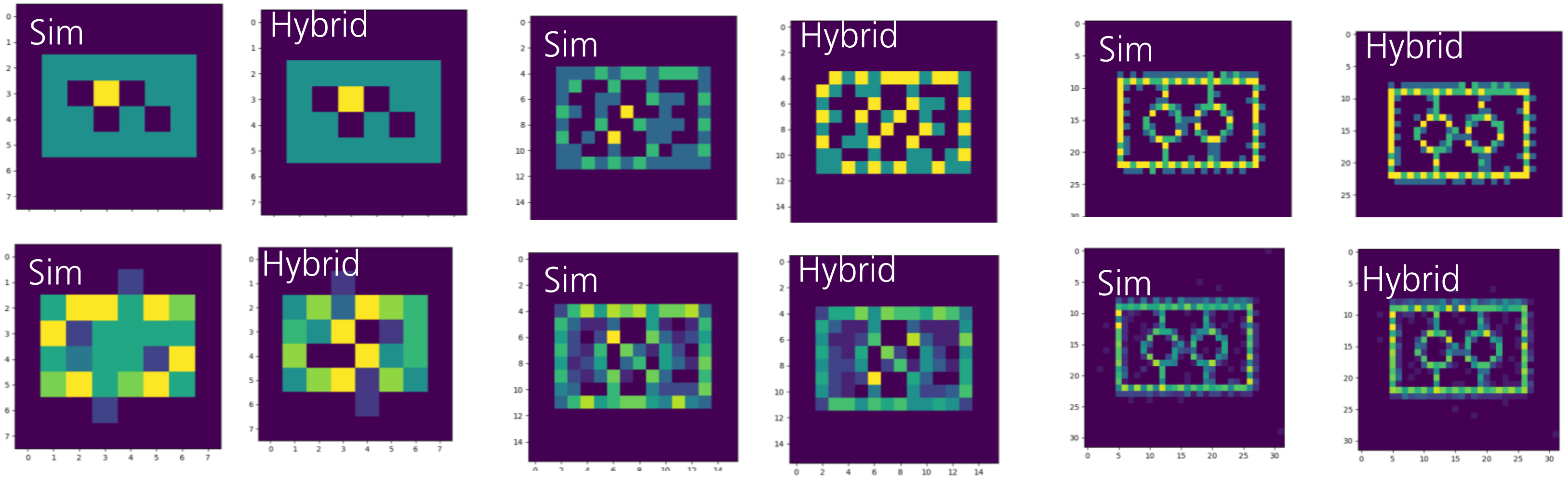
32x32
64 projections

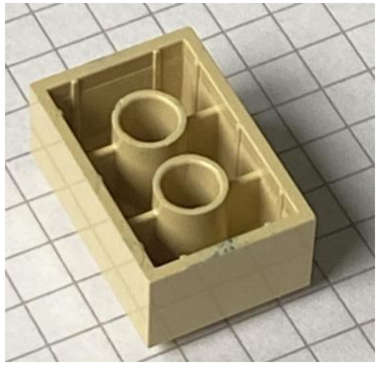
FBP



2-bit

4-bit





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Fraunhofer EZRT

Optimization of CT Data Acquisition by means of Quantum Computing

Summary & Discussion

Optimization of few projections trajectory

- we developed & implemented an algorithm to optimize the selection of trajectories for Computed Tomography on a real-world Quantum Computer

- the stochastic complexity of the task increases quickly with higher numbers of projections.

  Thus, to solve the task of optimization via a QUBO algorithm on a QC is expected to show advantages of quantum computing for a real-world problem

CT image reconstruction

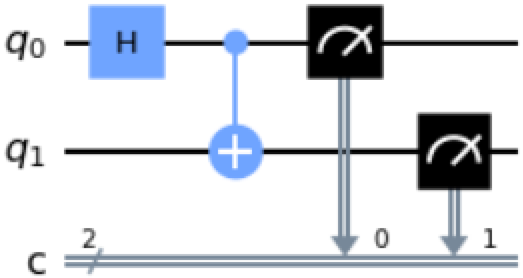
- image reconstruction for industrial CT was performed on a QC-device. Nevertheless, today image size and pixel dynamics are still very limited.

  We expect to improve the QC-devices rapidly – reaching usual image dimensions in foreseeable time

Research on these topics will continue…!

Optimization of CT Data Acquisition by means of Quantum Computing

The End



Thank you for your attention!

11th Conference on Industrial Computed Tomography, Wels, Austria (iCT2022):
R. Schielein et al. „Quantum Computing and Computed Tomography: A Roadmap towards QuantumCT"

12th Conference on Industrial Computed Tomography, Fürth (iCT2023):
T. Lang, S. Semmler, et al. „N-Dimensional Image Encoding on Quantum Computers »

13th ECNDT 2023 - European Conference on Non-Destructive Testing (ECNDT2023), Lissabon July 3rd – 5th, 2023:
T. Fuchs et al. „Optimization of Computed Tomography Data Acquisition by means of Quantum Computing"

13th Conference on Industrial Computed Tomography (iCT2024), February, 6th- 9th 2024, Wels:
D. Prjamkov et al. „Comparison of Different Quantum Computing Devices for Optimization of Computed Tomography Data Acquisition"