

# The physical meaning of the autocorrelation function in NDT thanks to signal processing

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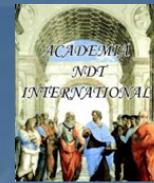
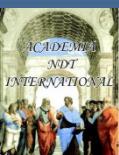
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## Physical interpretation of the correlation function

Convolution equation

$$y(t) = c(t) * h(t) = \int_{\mathbb{R}} h(t - t')c(t')dt',$$

Autocorrelation function

$$\gamma_{xx}(\tau) = \int_{\mathbb{R}} x(t)x^*(t - \tau)dt,$$

Google autocorrelation of white noise

# Content

- Introduction : introduction to signal processing
  - Pulse inversion as an example for extracting nonlinear signatures
  - Principle of Time Reversal (TR) based NEWS techniques
    - Future trends for medical imaging and NDT (Non Destructive Testing) of complex media
  - The use of coded excitation in order to access to the correlation properties
- Methods and materials
  - Complex medium for reverberating and long time properties
  - Linear signal processing : the physical meaning of the cross correlation function (memory aspects)
  - Symmetry Analysis, representation and memory based /nonlinearity signatures
- Conclusions, discussion and perspective

# Academia NDT Signal Processing Chapter

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## Signal Processing for Non Destructive Testing (NDT)

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**Abstract.** A review of modern signal processing methods is suggested. All standard NDT methods are described from the signal processing point of view, beginning from historical ideas and systems, and ending with promising modern approaches.

**Key words:** non destructive testing, signal processing.

### 1 Introduction

Chapter supported by

- V. Vengrinovich
- B. Raj
- Z. Prevorovsky
- W. Rummel
- and others members ...

Signal processing : a « new » area (Shannon, 1948) compared to mathematics, physics, medicine, chemistry, ...

# Excitation of Nonlinear Systems : concept

## ■ Linear systems



- output spectrum properties are « invariant » with respect to excitation
- lots of invariants including scaling effects, reciprocity and time reversal

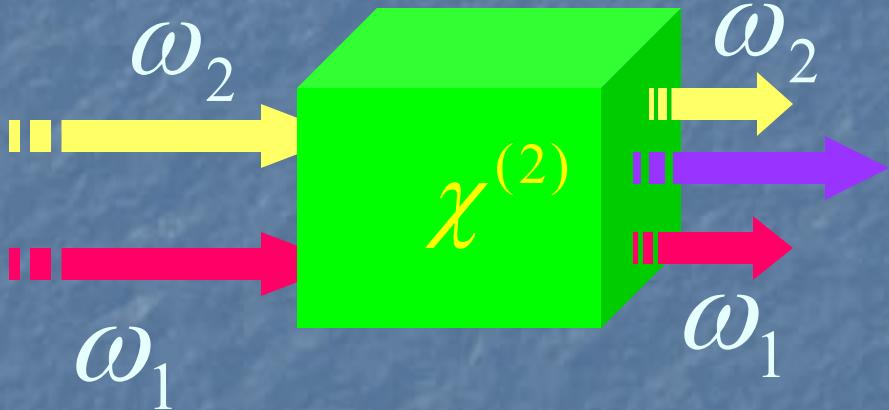
## ■ Nonlinear systems

- spectrum is modified : spectrum representation is not an « invariant »
- is it still interesting to look at frequency components ?
- what is the next “invariant” candidate instead of sine wave excitation?
  - time evolution of frequency representation : wavelets and second order tools ...



- It depends on the system
- how to find such invariant ?

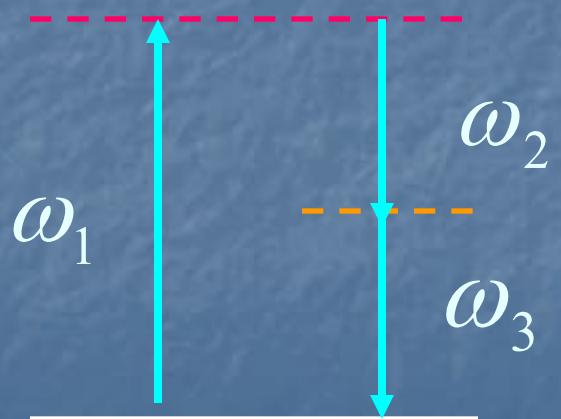
# Nonlinear optics : generation of intermodulation products



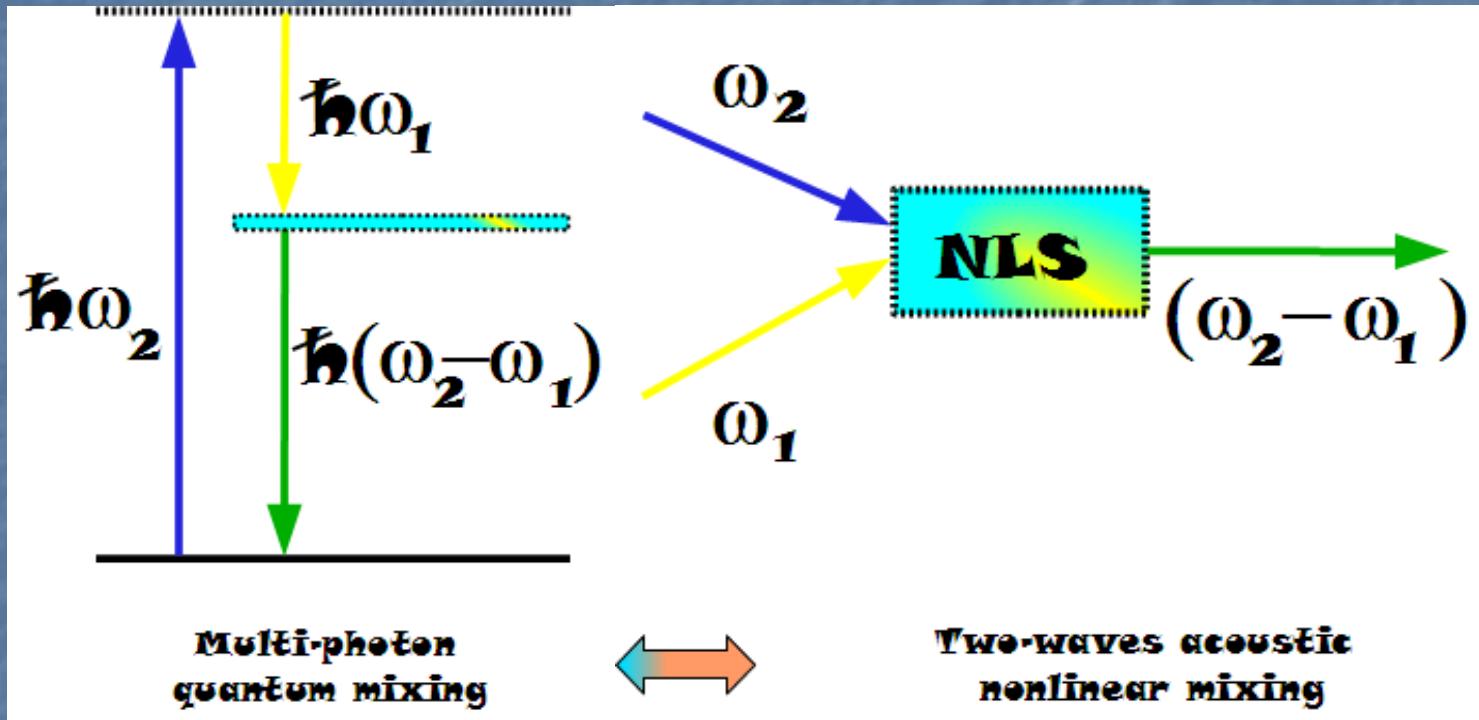
$$\omega_3 = \omega_1 - \omega_2$$

**Applications:**

**Parametric amplification**



# Representation of the nonlinear mixing of waves (optic/acoustic)



(S. Dos Santos, [Academia NDT International meeting, Eger, April 2011](#))

- Symmetries and Lie groups
  - Symmetries and Lie groups
  - Optimisation of the excitation

# Excitation of Nonlinear Systems : experiments

- Linear systems (amplitude is not critical)
  - time domain : pulse
  - frequency domain : sine waves are eigen-functions
- Nonlinear systems (amplitude is critical)
  - time domain : pulse amplitude must be known (calibration)
  - frequency domain : sine waves are not eigen-functions (modulation)
  - attenuation and frequency are time-dependant (slow dynamic)
  - scaling effects : how to take into account them systematically
    - It depends on the system
    - how to find such excitations ?



Every  
particular  
problem needs  
a specific tool



Nonlinear Signal Processing

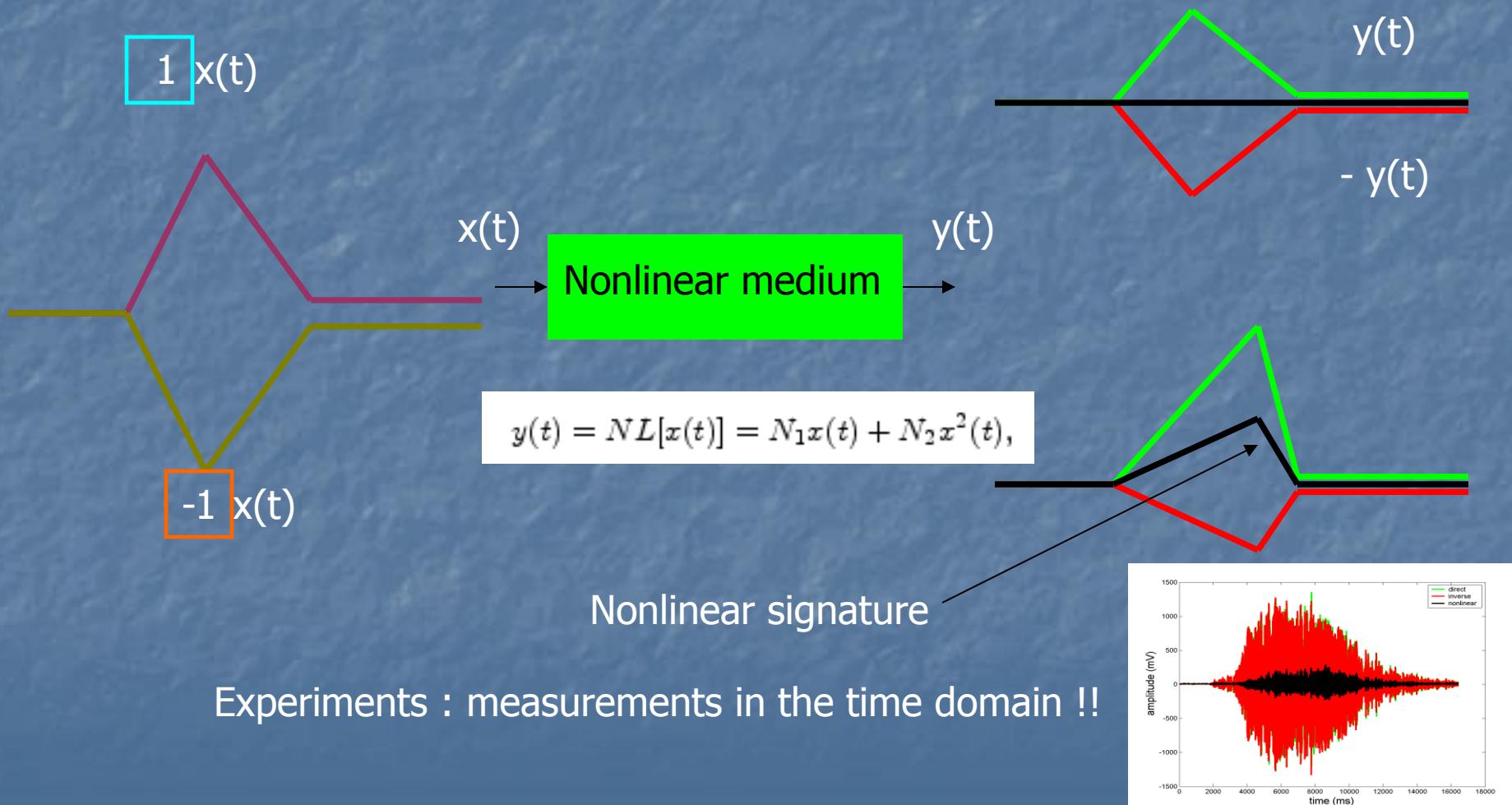
# A specific tool for each NL signature

- harmonics generation
- intermodulation
- modulation, auto-modulation
- amplitude dependence of “classical linear signatures”
  - resonance frequency
  - attenuation
- sub-harmonics
- low frequency effects <-> slow dynamics
- chaos ...
  
- Generic signature in various physical systems
  - mechanics, optics, electronics, acoustics, control, robotic, and NDT

nonlinearity level



# Nonlinear Signal Processing : Symmetrization of Excitation with Pulse Inversion (PI)



# Higher order Pulse Inversion (PI) method : ESAM

$$y(t) = NL[x(t)] = N_1 x(t) + N_2 x^2(t) + N_3 x^3(t),$$

C3 character table and irreducible representation

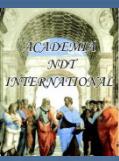
$C_3$	$E$	$\epsilon$	$\epsilon^*$	$C_3$	$E$	$\epsilon$	$\epsilon^*$
$E$	$E$	$\epsilon$	$\epsilon^*$	$A_1$	1	1	1
$\epsilon$	$\epsilon$	$\epsilon^*$	$E$	$A_2$	1	1	-1
$\epsilon^*$	$\epsilon^*$	$E$	$\epsilon$	$E_1$	2	-1	0

MULTIPLICATION TABLE (LEFT) AND CHARACTER TABLE (RIGHT) FOR THE POINT GROUP  $C_3$ .  $E$  IS THE IDENTITY,  $\epsilon = e^{\frac{2i\pi}{3}}$  DENOTES ROTATION BY AN ANGLE  $\frac{2\pi}{3}$ ,  $\epsilon^* = e^{-\frac{2i\pi}{3}}$  DENOTES ROTATION BY AN ANGLE  $-\frac{2\pi}{3}$ .



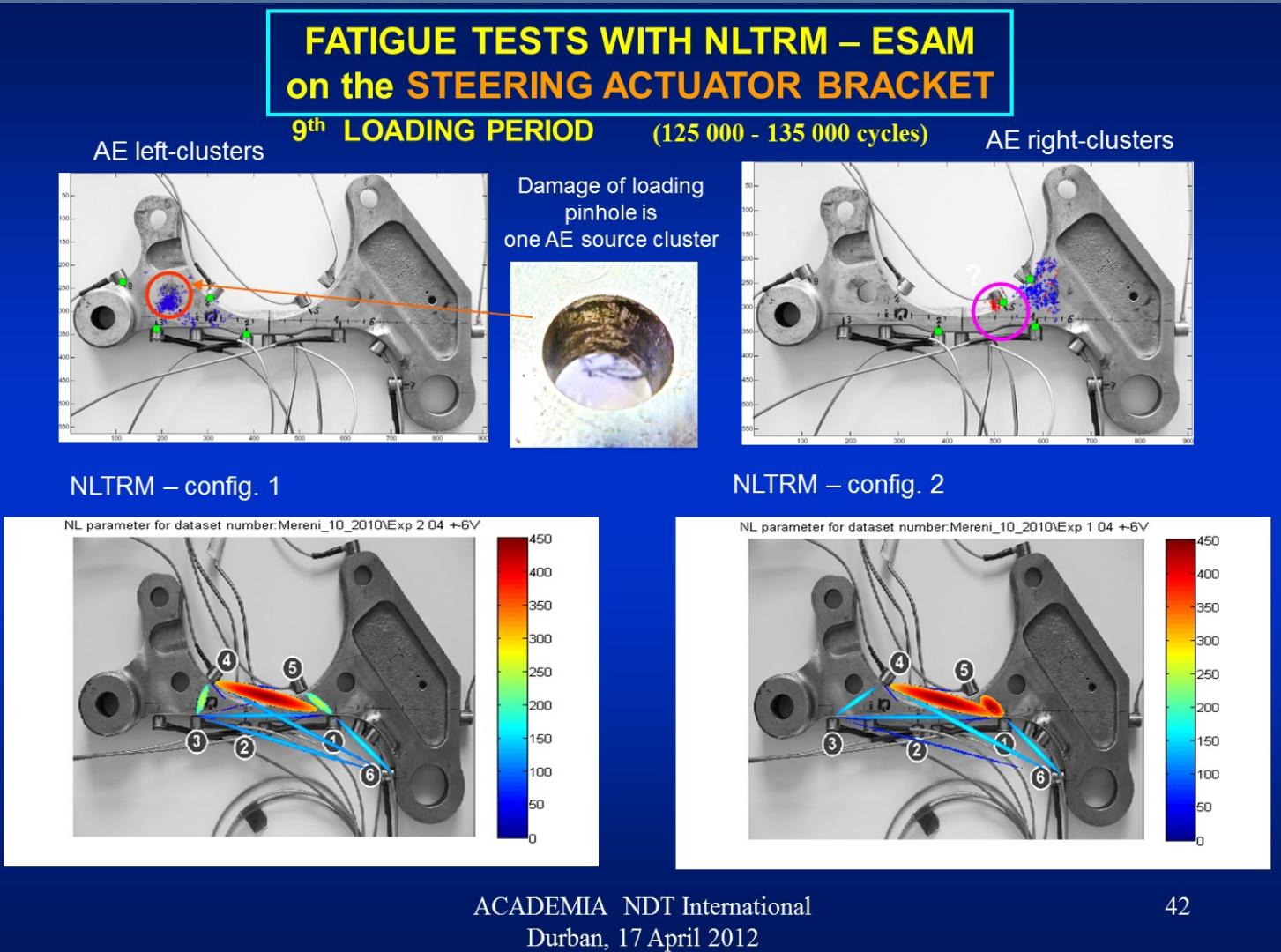
New « symmetrized » excitations

S. Dos Santos and C. Plag. *Excitation symmetry analysis method (ESAM) for calculation of higher order nonlinearities*, International Journal of NonLinear Mechanics, 43 :164–169, 2008



# Technical validations in NDT

# Application of ESAM signal processing



Nonlinear ultrasonic time reversal mirrors in NDT, Zdenek Prevorovsky, Czech Academy of Science, Academia NDT lecture at WCNDT, Durban (2012)

# Group Theory and Signal Processing

## NLTRM - ESAM

S. Dos Santos, C. Plag, "Excitation symmetry analysis method (ESAM) for calculation of higher order nonlinearities"  
Int. J. of Non-Linear Mechanics, 43, 164–169, 2008.

### ESAM – Excitation Symmetry Analysis Method

Based on symmetrical analysis (properties of point group  $C^3$ )

Permits extraction of a nonlinear parameter – coefficient  $N_3$  from the nonlinear response  $y$  to the excitation  $x$ . The response is considered to be a 3<sup>rd</sup> order polynomial function of variable  $x$ :

$$y(t) = N_1 x(t) + N_2 x^2(t) + N_3 x^3(t)$$

3 Excitations corresponding to the irreducible representations of point group  $C^3$  are used for nonlinear part extraction :

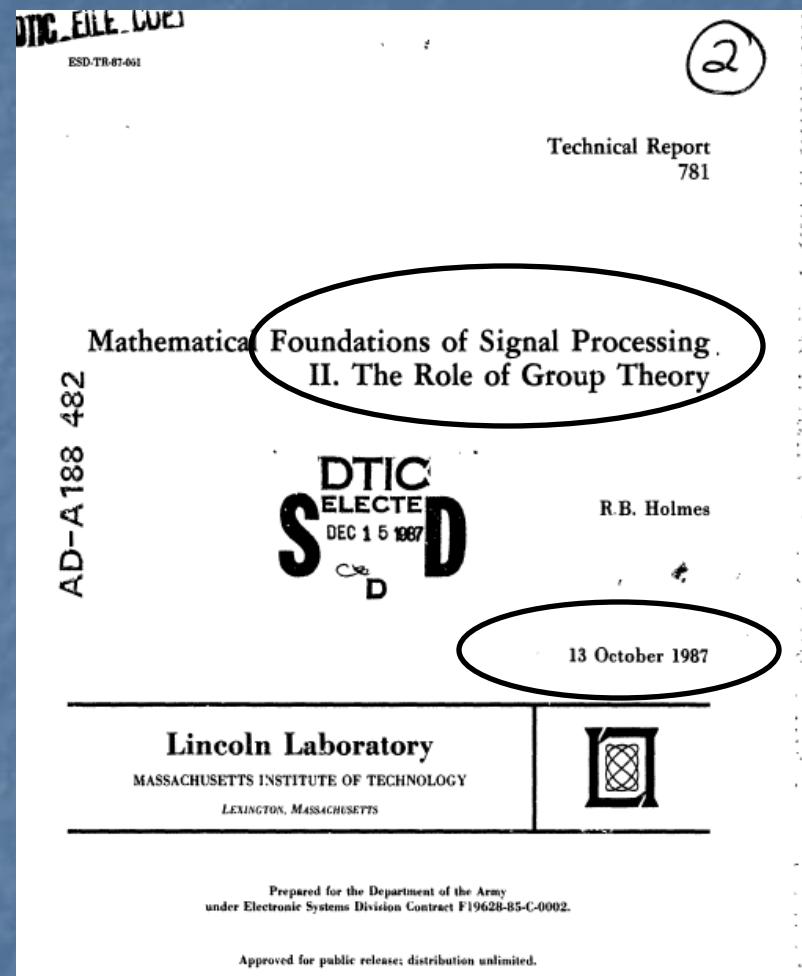
$$x_E(t) = x(t) \quad x_{\varepsilon}(t) = x(t) \cdot e^{\frac{2i\pi}{3}} \quad x_{\varepsilon^*}(t) = x(t) \cdot e^{-\frac{2i\pi}{3}}$$

Response  $y$  is measured at each excitation  $x$  :

$$y_E(t) \quad y_{\varepsilon}(t) \quad y_{\varepsilon^*}(t)$$

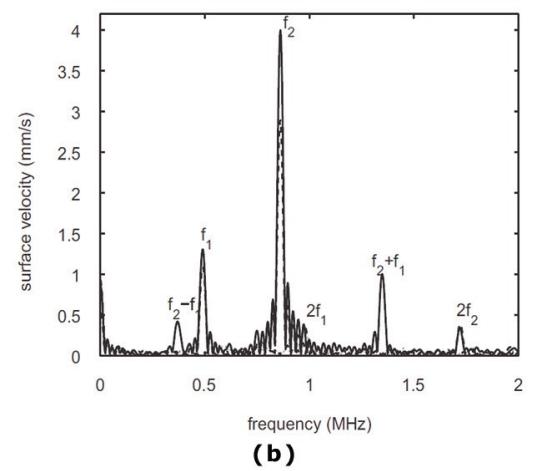
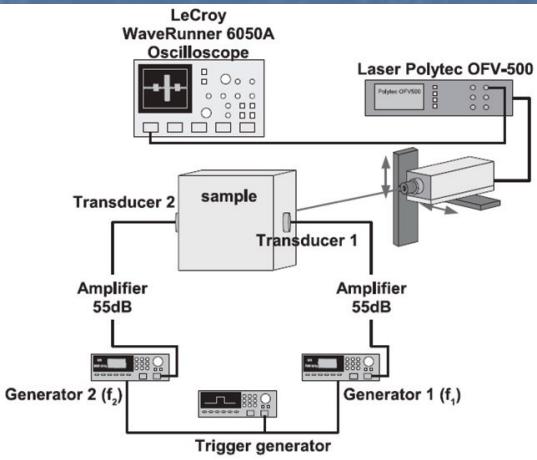
ACADEMIA NDT International  
Durban, 17 April 2012

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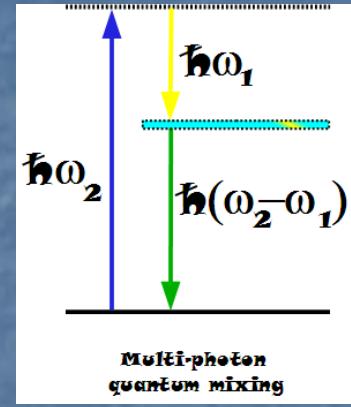
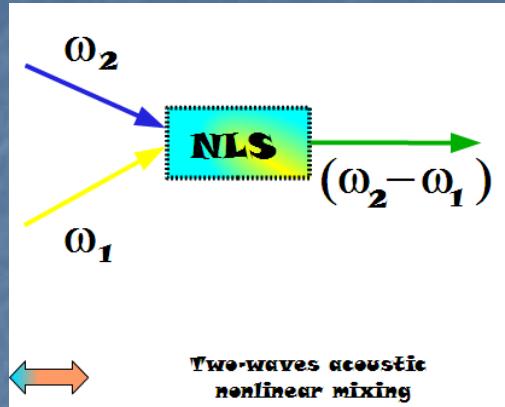


[1] S. Dos Santos, S. Vejvodova and Z. Prevorovsky, "Nonlinear signal processing for ultrasonic imaging of material complexity"; Proceedings of the Estonian Academy of Sciences, Vol. 59, Issue 2, pp. 108–117 (2010), <http://www.kirj.ee/proceedings?id=17029>

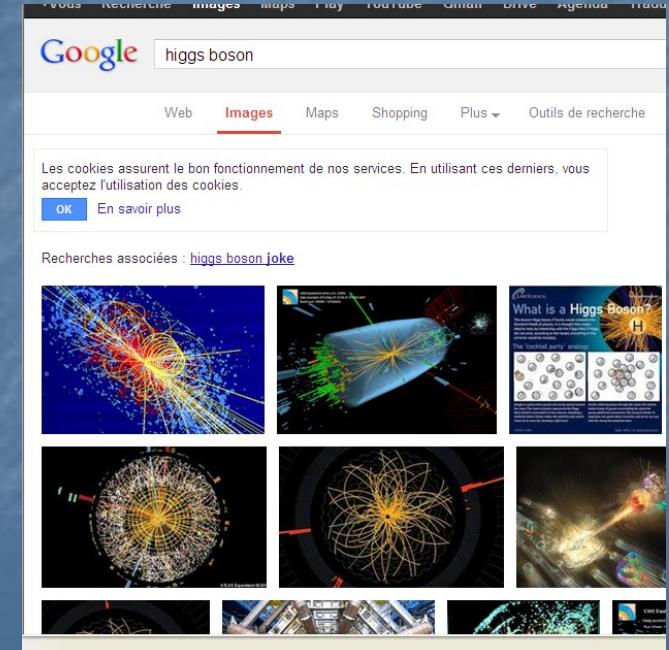
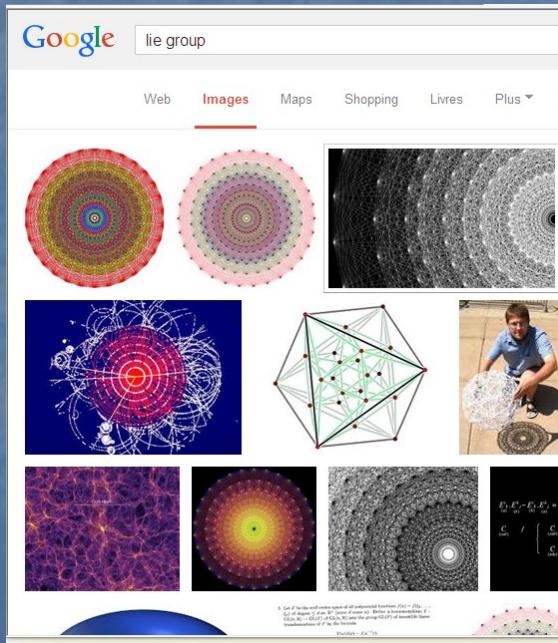
# Group Theory, Signal Processing and NDT



Nonlinear Mixing,  
*Lectio Materia, Academia NDT*



Nonlinear  
Signal Processing  
using Group  
theory  
representations



# Time Reversal (TR) for NEWS

M. Fink , France  
IEEE Trans on UFFC (1992)

K. Van Den Abeele, P.A. Johnson, and A. Sutin.  
Res. Nondestr. Eval. (2000)

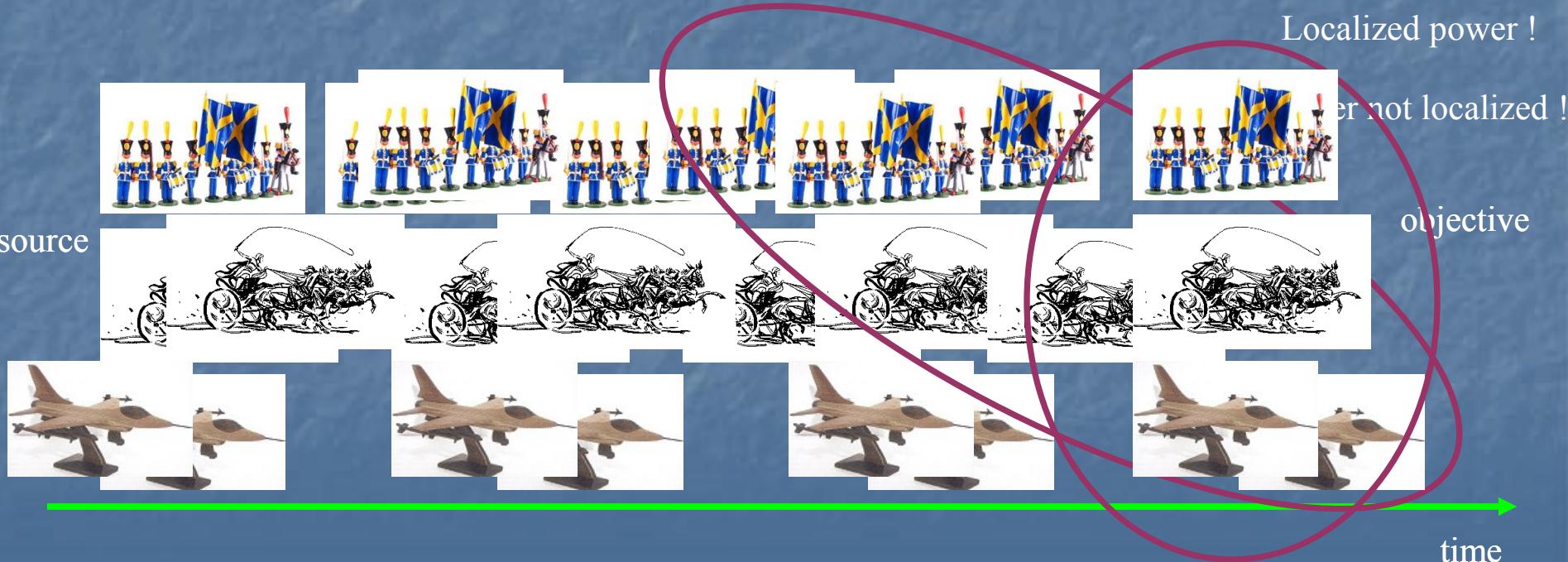
- NEWS : Nonlinear Elastic Wave Spectroscopy
  - What is the nonlinear signature due to **damaged** area or **aged** media ?
  - A high level of ultrasound is needed
  - How to **activate** or/and **localize** sources of nonlinearity ?
- Time Reversal (TR)
  - Signal processing method using correlation and convolution methods
  - Retrofocused signal with high level of ultrasound (for NL effects)
  - Temporal focusing : reconstruction of coherent tone-burst signals
  - Spatial focusing : analysis on localized point : the focused region  
(practically measurements can be done with laser interferometers )

Van Den Abeele and Dos Santos, 2006 (in french) <http://www.bruit.fr/flipbook/AT45/files/assets/basic-html/page26.html>

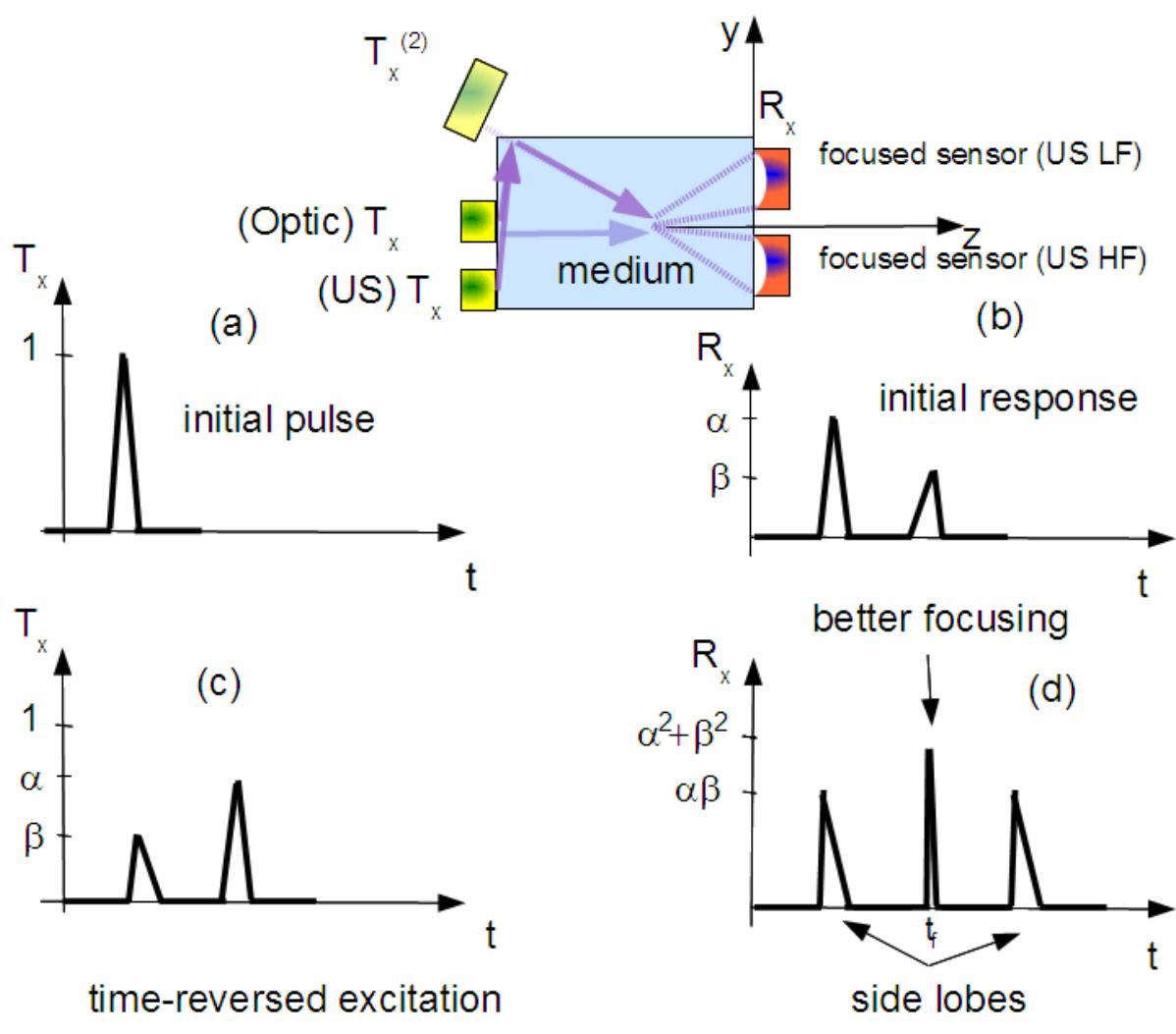
T.J. Ulrich *et al* , 2008 (in english) <http://www.sciencedirect.com/science/article/pii/S002074620700234X>

# Nonlinear Acoustics and TR

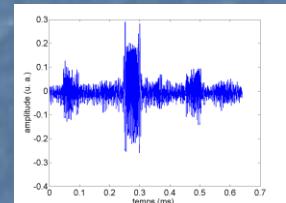
- TR : Time Reversal (M. Fink, France). It combines advantages :
  - of increasing the acoustic pressure stress localization
  - The local evaluation of the medium (which could be highly nonlinear) using global complex and **dispersive** response of the whole medium (which is mostly linear)
- It assumes a powerfull symmetry between sources and receivers : reciprocity



# TR-NEWS in NDT



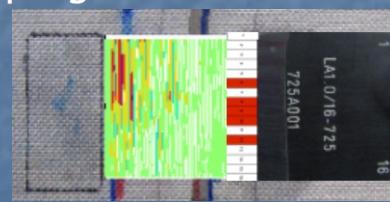
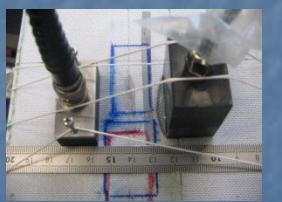
Dos Santos *et al*, French Congress of Acoustics, 2006



Le Bas *et al*, ECNDT 2006 Berlin



Dos Santos *et al*, NDT in progress 2009



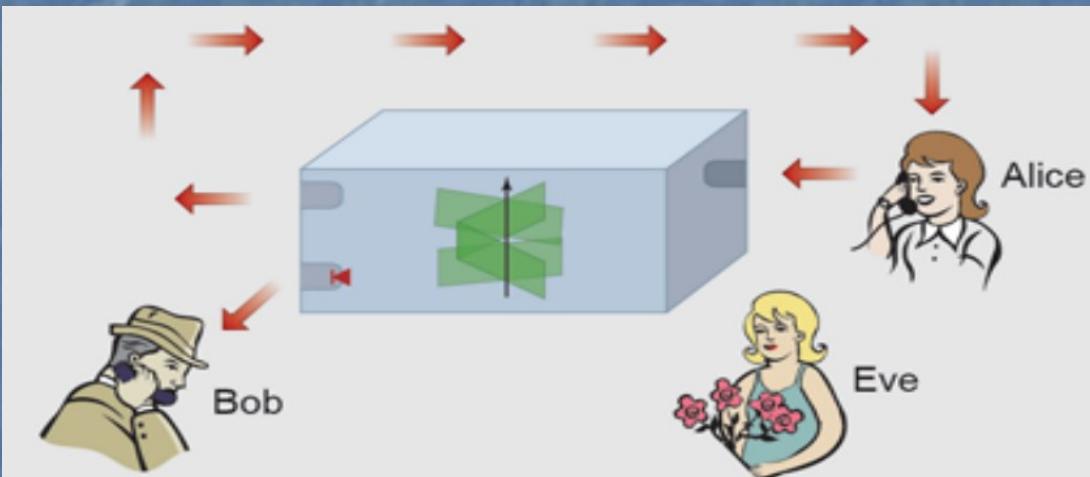
sub-surfacic imaging

Multi-modality using TR-NEWS : the objective here with FTIR

# Nonlinear Time Reversal for localization of nonlinear sources

"*Nonlinear Time Reversal in a Wave Chaotic System*", par Matthew Frazier, Binyam Taddese Thomas Antonsen and Steven M. Anlage,  
[Phys. Rev. Lett., 110, 063902 \(2013\)](#)

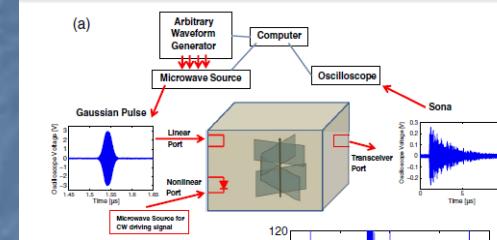
Nonlinear Time Reversal in a chaotic system for extraction of localized nonlinear electromagnetic signatures



Synopsis: Alice and Bob Go Nonlinear APS/Carin Cain

Perspectives : symbiosis between Time Reversal Techniques, chaotic waves, nonlinear dynamics and NDT of local activity

New area of Research and New innovations : cryptography and secured and coded communications



T. Goursolle, S. Dos Santos, O. Bou Matar, and S. Callé, [Non-linear based time reversal acoustic applied to crack detection: Simulations and experiments](#), Int. J. Non-Linear Mech. 43, 170 (2008)

# Coded Signal processing from Medical applications

and

# Symmetry Analysis

Dos Santos S. and Chaline J., *Symmetry analysis for nonlinear time reversal methods applied to nonlinear acoustic imaging*, AIP Conference Proceedings, 1685, 040017 (2015), DOI: <http://dx.doi.org/10.1063/1.4934412>

# Symmetry associated to chirp-coding



- linear response of systems

$$\boxed{-1} \quad y(t) = c(t) * h(t) = \int_{\mathbb{R}} h(t - t') c(t') dt', \quad \boxed{-1}$$

- chirp-coded response and correlation

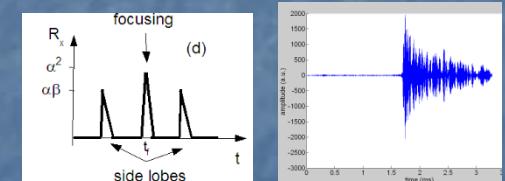
$$\boxed{1} \quad \Gamma(t) = y(t) * c(-t) = h(t) * \boxed{-1} c(t) * \boxed{-1} c(-t)$$

$$\Gamma_c(t) = c(t) * c(-t) = \delta(t)$$

correlation  $\sim$  impulse response

TR-NEWS

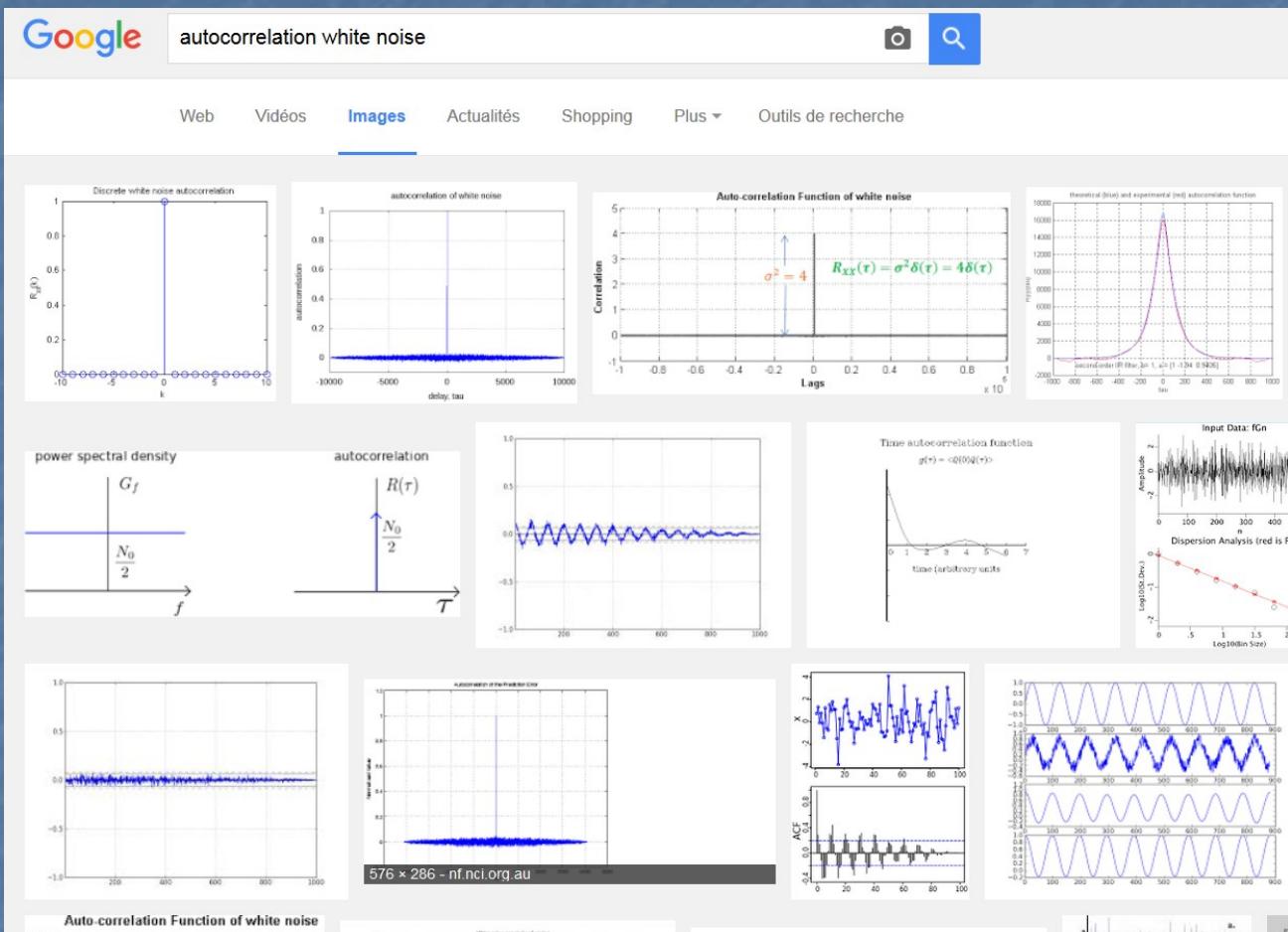
$$y_{TR}(t) = \Gamma(-t) * h(t) = \Gamma_h(-t),$$



Nonlinear signature extraction with Pulse Inversion : addition of responses

Nonlinear signature extraction with Chirp coded Pulse Inversion : substraction of responses

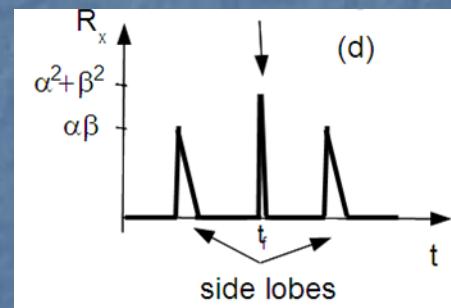
# What's the physical meaning of the white noise autocorrelation ?



Correlation function

$$\gamma_{xx}(\tau) = \int_{\mathbb{R}} x(t)x^*(t - \tau)dt,$$

Energy ?



Time

Symmetry wrt the max

# TR-NEWS : the physical meaning of the autocorrelation function

Convolution equation

$$y(t) = c(t) * h(t) = \int_{\mathbb{R}} h(t - t')c(t')dt',$$

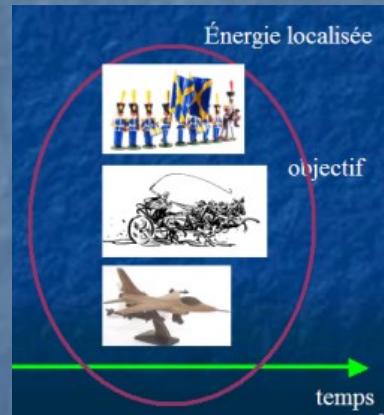
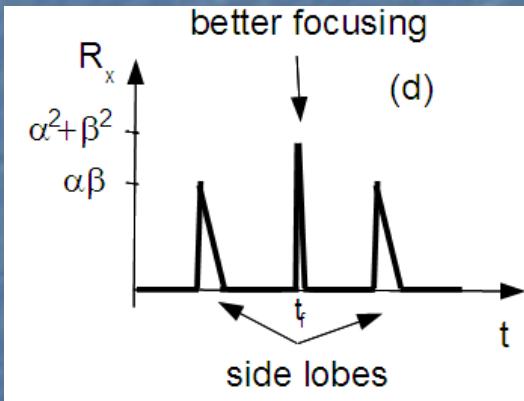
Correlation fonction

$$\gamma_{xx}(\tau) = \int_{\mathbb{R}} x(t)x^*(t - \tau)dt,$$

TR-NEWS



$$y_{TR}(t) = \Gamma(-t) * h(t) = \Gamma_h(-t),$$



TR-NEWS process is a way to understand the physical interpretation (energetically) of the autocorrelation function of a complex medium

# Properties of autocorrelation functions

- A narrow autocorrelation function generally implies a “broad” excitation
- and a “broad” autocorrelation function generally implies a narrow-band excitation
  
- In order to activate the spectrum broadening of the correlation function of a system, long time behavior with memory aspects should be activated

# Efficiency of TR-NEWS in complex medium : complexity and long time memories are advantages ! !

# Strategies for TR-NEWS focusing improvement and nonlinear measurements : chaotic transducer

Dos Santos *et al*,  
NDT in progress 2007

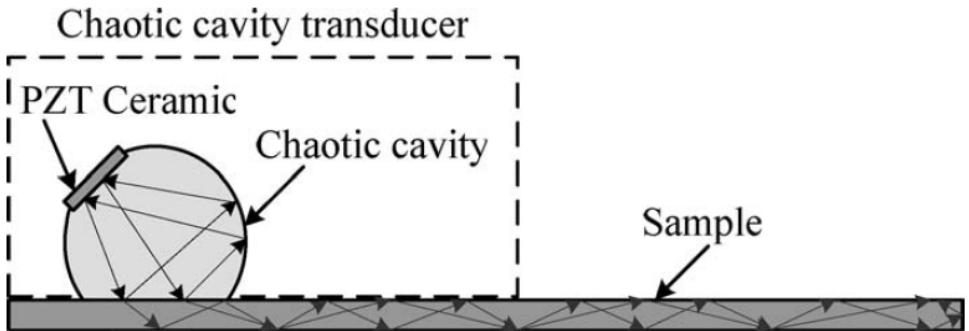
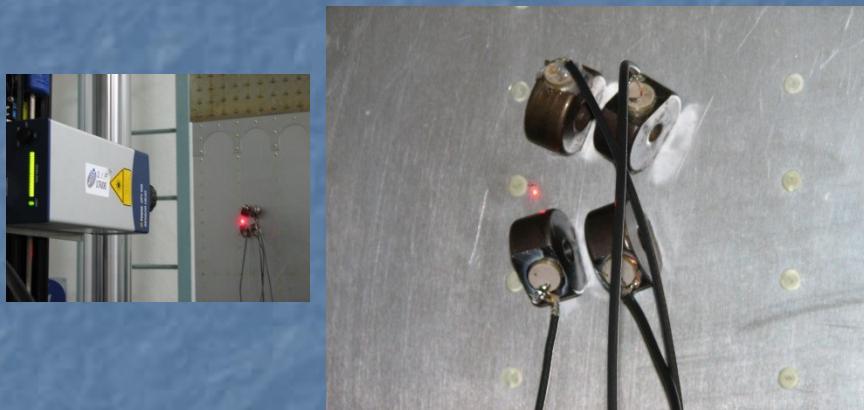


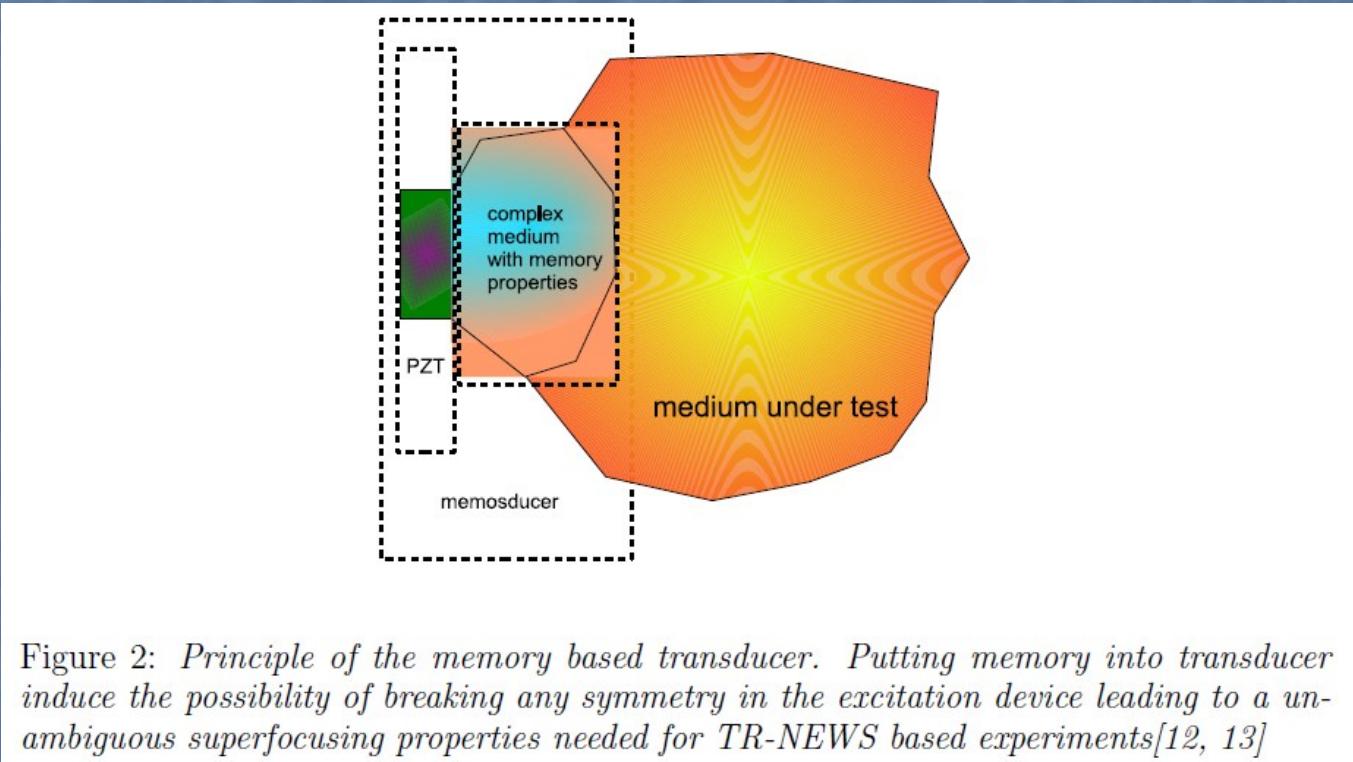
FIG. 1. Principle of the chaotic cavity transducer.



- For TR-NEWS based focusing, “long time coda” is needed :
  - Reverberant properties should be present in the “transducing device”
  - Chaotic cavities should be preferred in order to reduce “symmetry” effects

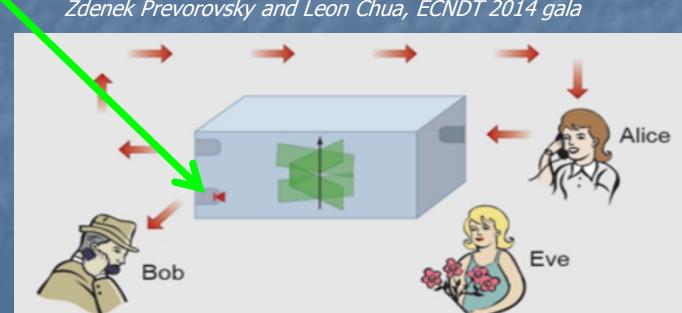
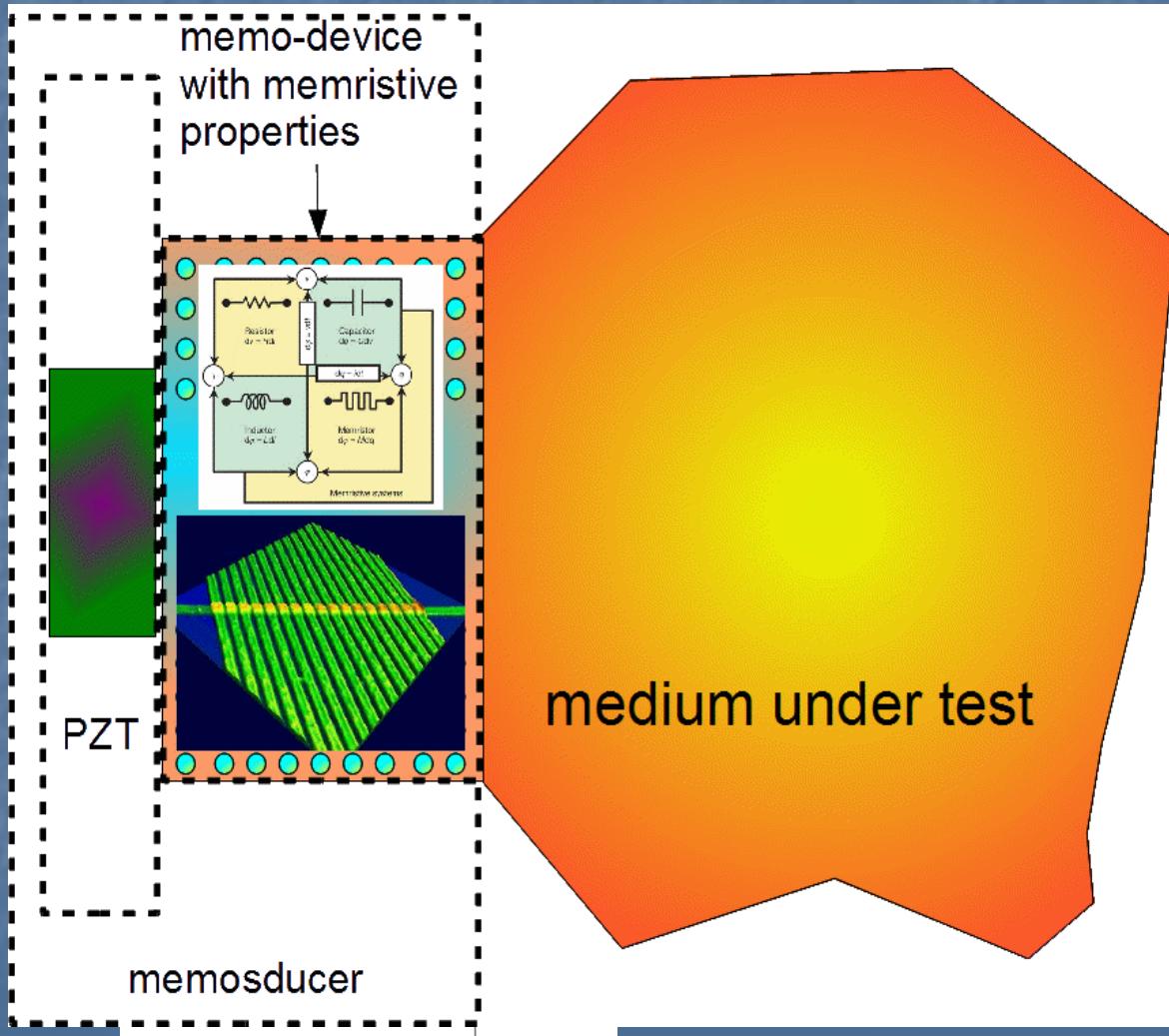
Bou Matar *et al*, *On the use of a chaotic cavity transducer in nonlinear elastic imaging*, Applied Physics Letters **95**, 141913 2009

# Strategies for TR-NEWS focusing improvement and nonlinear measurements : memristive transducer

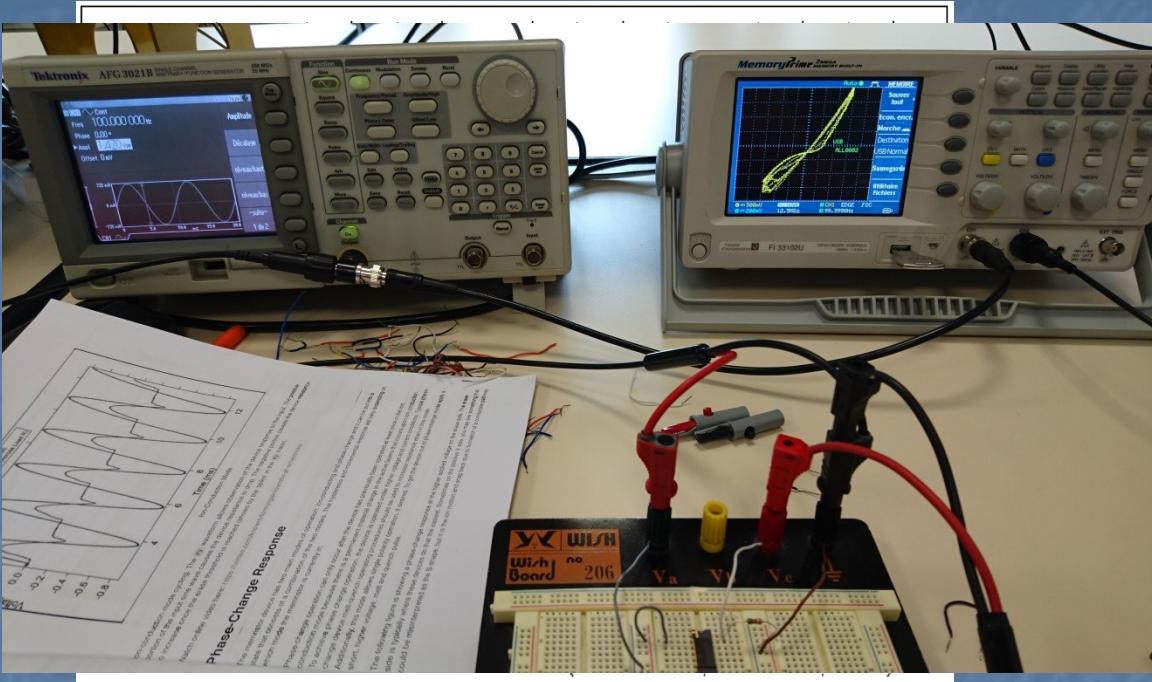
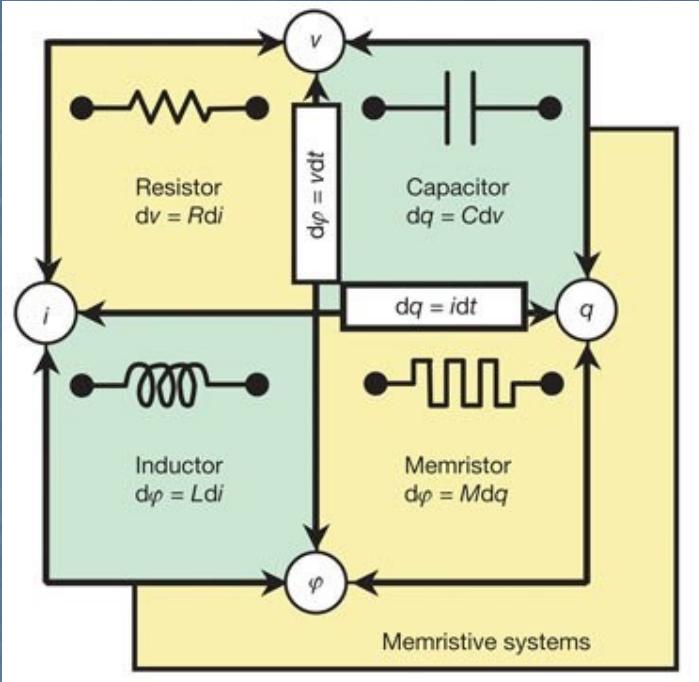


S. Dos Santos et al, proc of the [SPMS2010](http://gams.fjfi.cvut.cz/index.html) (<http://gams.fjfi.cvut.cz/index.html>) Stochastic and Physical Monitoring Systems, Decin, Czech Republic, ISBN 978-80-01-04641-8, pages 11–24, 2010

# Nonlinear Time Reversal and complex medium

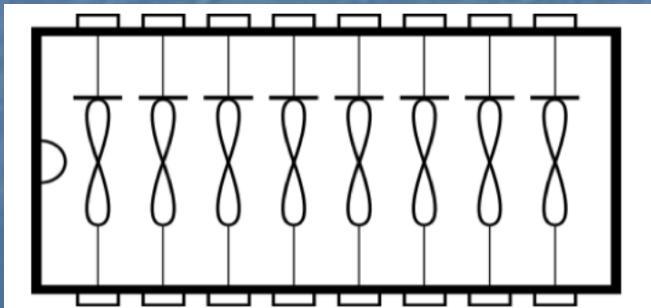


# The surprizing memristor chip



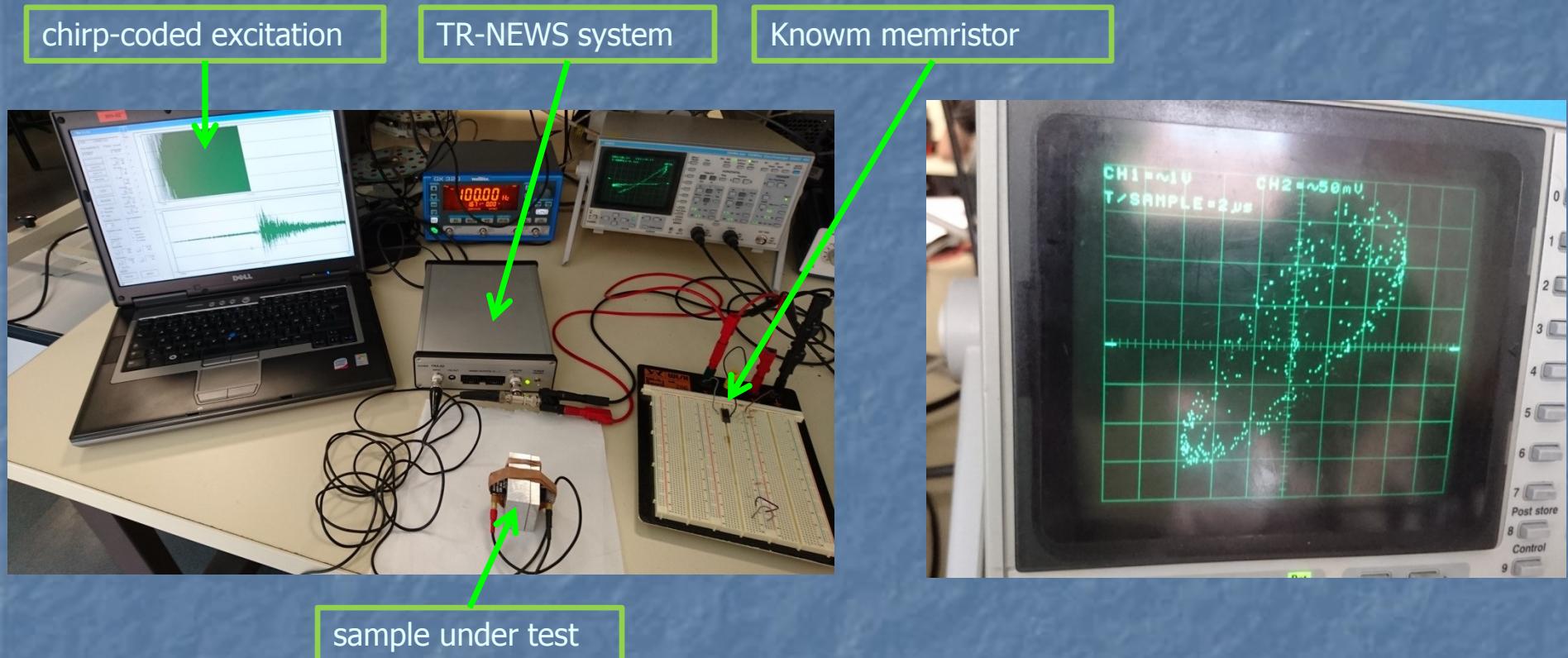
Chua, L. O., "Memristor-the missing circuit element," IEEE Transactions on Circuit Theory, vol. 18, no. 5, pp. 507-519, 1971

Strukov, D.B., Snider, G. S., Stewart, D. R. and Williams, R. S. "The missing memristor found," Nature 453, pp. 80-83, 2008



<http://knowm.org/>

# The memosducer for TR-NEWS

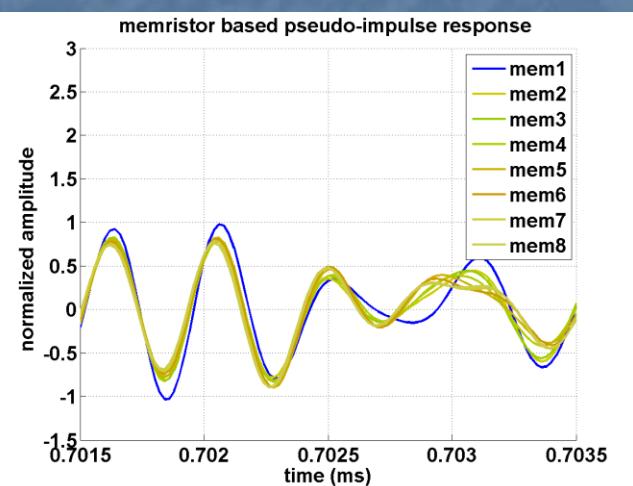
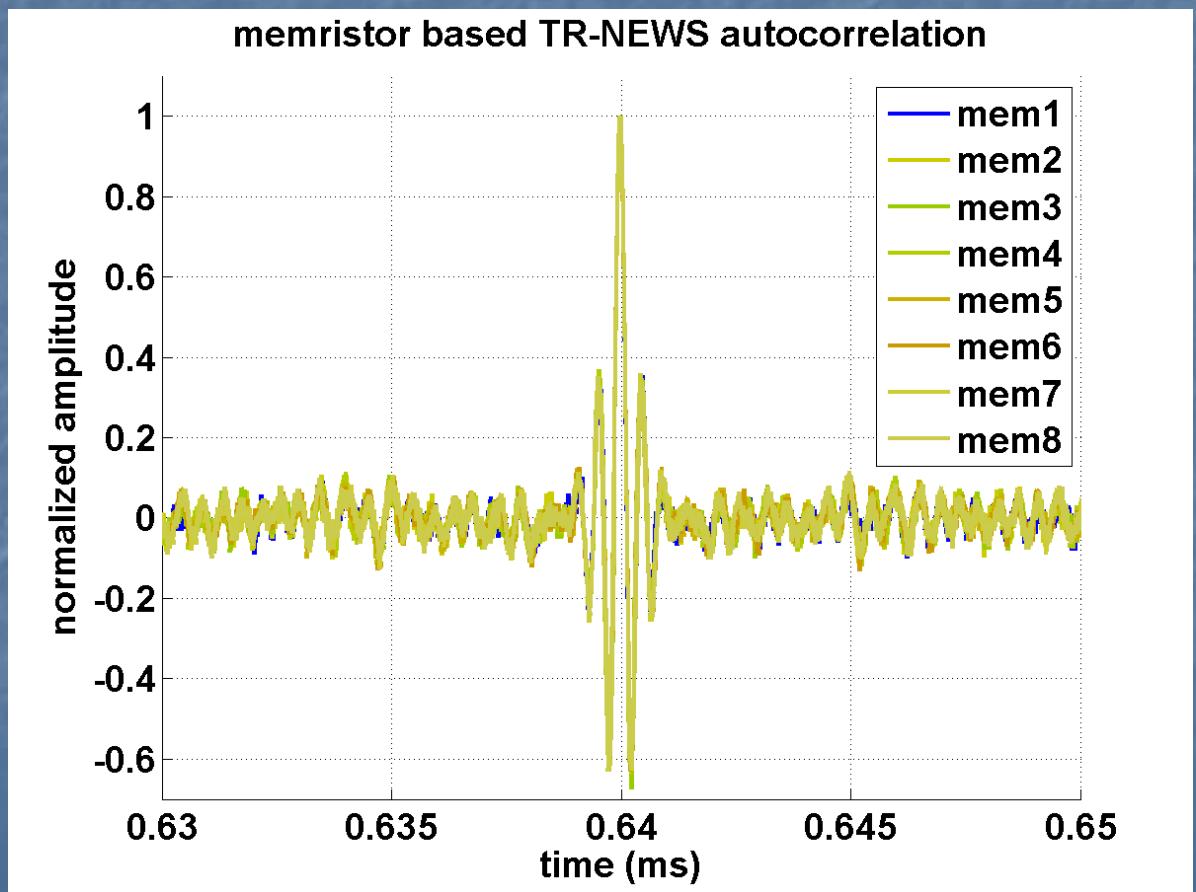
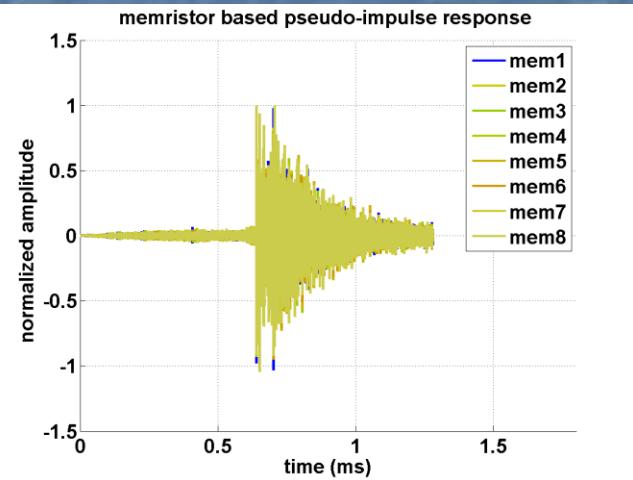


The memristor is inserted in order to change the transducing process during ultrasonic testing (UT) of bounding in aluminum sample

During the chirp-coded excitation, the memristor properties are activated in order to generate complex behavior like a chaotic cavity

*A memristor based ultrasonic transducer : the memosducer*, S. Dos Santos and S. Furui, to appear in the proc of the 2016 IEEE IUS Symposium, Tours, France

# The memristor autocorrelation



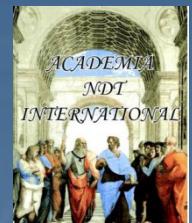
# Conclusion and future work

## ■ Conclusion

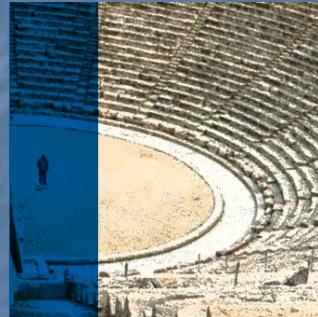
- The physical interpretation of correlations functions has been presented
- The Pulse Inversion signal processing has been **generalized** within Symmetry Analysis formalism
- Extraction of nonlinear signature can be **enhanced** using Symmetry Analysis
- Improvement of TR-NEWS sensitivity with **optimized excitation** is validated by experiments in order to activate nonlinear signature and memory properties

## ■ Future work

- Standardization of nonlinear signature extracted with advanced signal processing
- Signal (and image) processing and information theory as the skeleton of modern NDT



## Announcements



**ICSV23**  
23<sup>rd</sup> International Congress on Sound & Vibration  
From Ancient to Modern Acoustics

10-14 July 2016      Athens, *Greece*   
Athenaeum Intercontinental Hotel

<http://icsv23.org/>

 2016 IUS INTERNATIONAL ULTRASONICS SYMPOSIUM



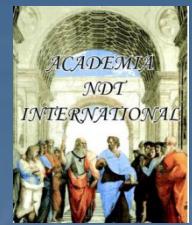
FRANCE TOURS



September 18-21, 2016, TOURS, FRANCE

IEEE

<http://sites.ieee.org/ius-2016/>



# Thank you !



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